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On the Friedman Rule in Search Models with Divisible Money

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Abstract

This paper studies the validity of the Friedman rule in a search model with divisible money and divisible goods in which the terms of trades are determined endogenously. We show that ex post bargaining generates a holdup problem similar to the one emphasized in the labor-market literature. Buyers cannot obtain the full return that an additional unit of money provides to the match, which makes the purchasing power of money inefficiently low in equilibrium. Consequently, even though the Friedman rule maximizes the purchasing power of money, it fails to generate the first-best allocation of resources unless buyers have all the bargaining power.

KEYWORDS: Money, Search, Friedman rule

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1 Introduction

This paper studies the validity of the Friedman rule in search-theoretic models with fully divisible money and a degenerate distribution of money holdings. In the absence of distorting taxes and search externalities, these models generate contradictory conclusions regarding the ability of the Friedman rule to guarantee an efficient allocation of resources. In Shi (1997, 1999, 2001) and in Berentsen and Rocheteau (2003) the Friedman rule generates the first-best allocation. In contrast, in Rauch (2000) and Lagos and Wright (2001) the Friedman rule only generates a second-best allocation, unless buyers have all the bargaining power, because the first-best would require deflation at a greater rate than the Friedman rule, which is inconsistent with the existence of a steady-state monetary equilibrium.[1]

In order to shed light on what the Friedman rule can accomplish in a search environment with fully divisible money, we modify Shi's (1997, 1999, 2001) framework in the following way. As in Shi's framework, we assume that the economy is populated by households consisting of a large number of members that pool together their money holdings after trading, which renders the distribution of money holdings degenerate.[2] In contrast to Shi's approach, but as in Rauch (2000), we assume that the bargaining strategies take into account the money holdings of the traders in a match. In comparison with Rauch (2000), our analysis has three distinctive features. First, we consider alternating offer bargaining games to determine the quantities traded in each match. The sequential bargaining procedure, in contrast to the axiomatic Nash bargaining solution, makes clear how the agents' behavior in out-of-equilibrium matches prevents the economy from attaining the first best under the Friedman rule. Second, we consider how buyers' bargaining power affects the purchasing power of money. Third, we allow for money hoarding by letting households choose the amount of money their members carry to the search market. This eliminates stationary monetary equilibria where the gross growth rate of the money supply is smaller than the discount factor.

The following results emerge from our paper. First, the frameworks of Shi (1997, 1999, 2001) and Lagos and Wright (2001) have equivalent closed-form solutions. This result is noteworthy, because Lagos and Wright and Shi use different formalization strategies to obtain a degenerate distribution of money holdings.[3] Second, and more importantly, we show that the different results in the literature are due to the presence of one term in the envelope condition, which reflects how agents expect other agents to react in response to

[1] We restrict our attention to search models with degenerate distributions because it is well known that with nondegenerate distributions expansionary monetary policies can be beneficial (Deviatov and Wallace 2001; Molico 1997). For a survey on the optimum quantity of money, see Woodford (1990).

[2] The large-household assumption, extending a similar one in Lucas (1990), avoids difficulties that arise in models with a nondegenerate distribution of money holdings, and so allows for a tractable analysis of money growth and inflation. In a series of papers, Shi (1997, 1999, 2001) has explored the use of this assumption in search models of money.

[3] Lagos and Wright (2001) assume that, after the random-matching market closes, a Walrasian market opens in which agents can trade a homogeneous good for money. They show that if the traders have quasi-linear preferences for this good, all agents leave the Walrasian market with the same amount of money.

an out-of-equilibrium change of their money holdings.

All models have in common that in each period the households make two decisions: they determine the trading strategies for their members, where a trading strategy instructs a member which offer to make and which offers to accept (a reservation utility) in the bargaining process; and they choose with how much money their members enter the search market. The crucial feature of Shi's approach is that the trading strategies are not contingent on the money holdings of the potential opponents in bargaining. They only take into account the amount of money that agents hold in equilibrium. Because of this, when a buyer and a seller meet and the buyer's money holdings differs from what is expected in equilibrium, the seller's reservation utility is left unchanged.[4] In contrast, in Rauch (2000) and in Lagos and Wright (2001) the bargaining strategies are match-specific, i.e. they take into account the money holdings of the buyer in a match. In the symmetric equilibrium, the money holdings of all agents in the market are equal (the distribution of money holdings is degenerate), and so all agents make the same equilibrium offers and have the same reservation values. Nevertheless, a seller's reservation utility in an out-of-equilibrium match differs from the reservation utility in equilibrium.

The change of sellers' reservation utilities in out-of-equilibrium matches is the reason why in Rauch (2000) and in Lagos and Wright (2001) the economy cannot attain the first-best under the Friedman rule. Because of this change, a buyer who brings an additional unit of money into a match cannot extract the whole surplus that this unit provides to the match, which lowers the marginal value of money. In this sense, money is an asset whose holder — the buyer — is not able to capture its entire return. As pointed out by Acemoglu and Shimer (1999), this holdup inefficiency is very common in search models with ex post bargaining. Because the Friedman rule only compensates for the time impatience of agents, correction of this holdup inefficiency requires a *higher* rate of deflation than the rate of time preference, which is inconsistent with the existence of a steady-state monetary equilibrium.

The paper is organized as follows. Section 2 presents the model and its main assumptions. Section 3 derives the symmetric equilibrium and Section 4 identifies the crucial difference in the specification of the envelope condition of the two approaches. The Friedman rule is investigated in Section 5, followed by a discussion of our results in Section 6. Finally, Section 7 concludes.

2 The model

2.1 The environment

The environment is similar to that of Shi (1997, 1999, 2001). Time is discrete. The economy consists of a continuum of infinitely-lived households that specialize in consumption and production. There are $H \geq 3$ types of goods and H types of households. Households are

[4] A trading strategy in Shi's model is not a complete plan of actions that specifies what a member has to do in all circumstances. In this sense, the sellers are like vending machines that are programmed in advance to accept any offer that gives them some reservation value. See also our discussion of Shi's approach in Section 6.

uniformly distributed among types. A household of type k produces good k and consumes good $k+1$ (modulo H). Denote by $z \equiv \frac{1}{H}$ the single-coincidence-of-wants probability.

Each household consists of a continuum of members, normalized to one, who carry out different tasks but regard the household's utility as the common objective. In order to avoid any incentive problems by the members, we assume that each household knows what happens to its members on the search market (who matches with whom, the bargaining process etc.) so it can punish members who fail to carry out the proposed equilibrium strategy.[5] We focus on a representative household, which we call household h. Decision variables of household h are denoted by lowercase letters. Capital letters denote other households' variables. Furthermore, variables corresponding to the next period are indexed by $+1$, and variables corresponding to the previous period are indexed by -1.

At the beginning of each period, household h holds m units of money. These m units of money can be either hoarded or distributed across members in order to be spent on the search market. Let $y \leq m$ be the number of units that each member carries in a match. In search models with divisible money it is usually assumed that agents bring all their money holdings into the matches, i.e., $y = m$. An explicit hoarding decision (choice of y) ensures that no monetary equilibrium exists if the rate of growth of the money supply is smaller than what is prescribed by the Friedman rule ($\gamma < \beta$).

Once members are matched, the household instructs them on how to bargain. Since goods and money are perfectly divisible, agents can exchange any quantity of money and goods they wish, provided that the traded quantity of money does not exceed the money holdings of the buyer in the match. After trading, household members consume the acquired goods and then return home and pool together their receipts of money. At the end of a period, each household receives a lump-sum transfer of money τ, which can be negative. The gross growth rate of the money supply is $\gamma = M_{+1}/M$, where M is the money supply per household in period t.

For each member of household h, consumption of q units of good $h+1$ provides utility $u(q)$, where $u(.)$ is a twice continuously differentiable function with $u(0) = 0$, $u'(q) > 0$, $u'(0) = \infty$, and $u''(q) < 0$. Production of q units of good h provides disutility $c(q)$, where $c(.)$ is a twice continuously differentiable function with $c(0) = 0$, $c'(q) > 0$, $c'(0) = 0$, and $c''(q) > 0$. We assume that there exists $q^* < +\infty$ that satisfies $u'(q^*) = c'(q^*)$.

All through the paper we restrict our attention to values of q such that $u(q) - c(q) \geq 0$. The utility of the household is defined as the discounted sum of the consumption utilities of all its members minus their production costs. The discount factor for the household is $\beta \in (0, 1)$. Finally, if $V(m)$ denotes the lifetime expected utility of an household endowed with m units of money, the marginal value of money is $\omega = \beta V'(m_{+1})$.

[5] An alternative assumption, which is equivalent with respect to its implications for the model, would be to assume that all family members are robots, which act according to the program installed by the household instead of enjoying or suffering their own utility or disutility. Either specification rules out potential incentive problems members may have. For example, why an atomless seller is willing to suffer the disutility of production, as opposed to claim not meeting a buyer, when his tomorrow's money holdings do not depend on his action today.

2.2 Bargaining

Each agent in the market is endowed with money and with a production opportunity, and therefore each agent is a potential buyer and a potential seller. When two agents meet, the match is either a single-coincidence meeting, where one agent (the buyer) is a consumer of the good produced by the other agent (the seller), or a no-coincidence meeting. Terms of trade are determined in alternating-offer bargaining games. In contrast to Shi's approach, households condition their bargaining strategies on the money holdings of their bargaining partners. For this to be feasible, we assume that in a match the level of money holdings of each trader is common knowledge.

The marginal value of money of a household, however, is not observable. Nevertheless, in a symmetric equilibrium all households have the same marginal value for money, and in the bargaining they attribute to each other the value Ω that prevails in equilibrium. There is no obvious way to address the issue of which value for the marginal utility of money h's partners will attribute to h if h deviates from its equilibrium strategy, for instance by accumulating more money. Throughout the paper, we assume that in out-of-equilibrium matches the bargaining partners still attribute to each other the value Ω that prevails in equilibrium.[6]

Without loss of generality, we consider the bargaining between agent i, who is a representative member of household h, and a randomly chosen agent of another household, whom we will call agent j. Agent i's decision variables are denoted by lowercase letters, whereas decision variables of agent j are denoted by capital letters. Each period is divided into a large number of subperiods of length Δ. If, in a given subperiod, it is agent i's turn to make an offer and agent j rejects the offer, then agent j makes a counteroffer in the following subperiod. There is an exogenous risk of a breakdown of the negotiation each time an offer has been rejected. This breakdown risk differs for sellers and buyers. If the seller has rejected the buyer offer, the breakdown probability is $\theta\Delta$ with $0 < \theta \leq 1$. If the buyer has rejected the seller offer, the breakdown probability is $(1-\theta)\,\Delta$. Because the length of a subperiod is small, $\theta\Delta$ and $(1-\theta)\Delta$ are assumed to be smaller than one. Finally, we assume that members of the representative household h make always the first offer. This simplifies the exposition, but does not affect the results, because we will consider the bargaining game when Δ goes to zero, where the first-mover advantage vanishes.

Assume first that agent i is the buyer. Then agent i proposes the offer $\left(q^b, x^b\right)$, where $q^b = q^b\,(y, Y)$ is the quantity of goods produced by his partner in exchange for $x^b = x^b\,(y, Y)$ units of money, where the offer $\left(q^b, x^b\right)$ may depend on the money holdings of buyer i, y, and the money holdings of seller j, Y. If seller j accepts the offer, the acquired money x^b will add to j's household at the beginning of the next period, whose value today is Ωx^b.[7]

[6] If the marginal value of money Ω is common knowledge in a match, then we get the same result under symmetric bargaining. However, under asymmetric bargaining the model cannot be solved analytically because one derivative cannot be determined (see Section 6).

[7] To see why, suppose that the measure of a member is μ. Then for the household, the value of x additional units of money received by a member is $\beta\left[V\left(m_{+1}+x\mu\right)-V(m_{+1})\right]$. To express the value of x additional units of money for a member, we must multiply this quantity by the scale factor $1/\mu$. Because members are atomistic, we let $\mu \to 0$ to get $\lim_{\mu\to 0}\beta[V\left(m_{+1}+x\mu\right)-V(m_{+1})]/\mu = x\beta V'(m_{+1}) = x\omega$.

Any optimal offer must make seller j indifferent between accepting and rejecting the offer:

$$-c(q^b) + x^b\Omega = R^s \tag{1}$$

where $R^s = R^s(y, Y)$ is seller j's reservation value which may also depend on buyer i's and seller j's money holdings y and Y, respectively.

Assume now that agent i is the seller. Then agent i proposes the offer (q^s, x^s), where $q^s = q^s(y, Y)$ is the quantity of goods produced by his partner in exchange for $x^s = x^s(y, Y)$ units of money, where the offer (q^b, x^b) may depend on the money holdings of seller i, y, and the money holdings of buyer j, Y. To be optimal, seller i's offer (q^s, x^s) must satisfy

$$u(q^s) - x^s\Omega = R^b \tag{2}$$

where $R^b = R^b(y, Y)$ is buyer j's reservation value which may also depend on seller i's and buyer j's money holdings y and Y, respectively.

The reservation values of sellers and buyers satisfy

$$R^s = (1 - \theta\Delta)\left[-c(Q^s) + X^s\Omega\right] \tag{3}$$

$$R^b = (1 - (1-\theta)\Delta)\left[u(Q^b) - X^b\Omega\right] \tag{4}$$

According to (3), if with probability $1 - \theta\Delta$ there is no breakdown of the negotiation after a seller has rejected a buyer offer, the seller makes the counteroffer (Q^s, X^s) where $Q^s = Q^s(y, Y)$ and $X^s = X^s(y, Y)$ may depend on y and Y. The reservation value of a buyer (4) has a similar interpretation.

3 Symmetric Monetary Equilibrium

Now we can describe household h's choice problem. When the household determines the terms of trade, it is subject to two constraints. First, household members cannot spend more money than what they have:

$$x^b \leq y \tag{5}$$

Second, household members cannot ask for more money than what their bargaining partner holds:

$$x^s \leq Y \tag{6}$$

A household's trading strategy consists of the terms of trades (q^b, x^b) and (q^s, x^s), and acceptance rules for the offers (Q^b, X^b) and (Q^s, X^s) by other households. Let λ in $\mathbb{R}^+$ denote the Lagrange multiplier associated with constraint (5), $\pi \in \mathbb{R}^+$ the Lagrange multiplier associated with constraint (6), and $\phi \in \mathbb{R}^+$ the Lagrange multiplier associated with the constraint $y \leq m$. Taking the bargaining strategies of other households and the distribution of money holdings as given, in each period the household chooses $(m_{+1}, y, q^b, x^b, q^s, x^s)$

Thus, from the point of view of the household, $x\omega$ is a member's indirect utility of receiving x units of money in a match.

to solve the following dynamic programming problem:

$$\begin{aligned} V(m) &= \max_{q^b,q^s,x^b,x^s,y,m_{+1}} z\int \left\{u(q^b) - c(q^s) + \lambda\left(y - x^b\right)\right. \\ &\qquad \left. +\pi\left(Y - x^s\right) + \phi\left(m - y\right) + \beta V(m_{+1})\right\} dF\left(Y\right) \end{aligned} \quad (7)$$

subject to the constraints (1), (2), and

$$m_{+1} - m = \tau + z\int \left\{x^s - x^b\right\} dF\left(Y\right) \quad (8)$$

where $F\left(Y\right)$ is the distribution of money holdings across traders in the market and where the terms of trade $\left(q^b, x^b\right)$ and $\left(q^s, x^s\right)$ are functions of Y.

Equality (8) describes the law of motion of the household's money balances. The first term on the right-hand side is the lump-sum transfer of currency that the household receives each period. The second term specifies sellers' expected money receipts from selling minus the buyers' expected expenses from buying goods.

Throughout the paper we restrict our attention to symmetric equilibria where all trading partners of household h hold the same amount of money Y. In such a symmetric environment and if we use (1), (2), and (8) to eliminate x^b, x^s, and m_{+1} from (7), respectively, then household h solves the following maximization problem

$$\begin{aligned} V(m) &= \max_{\left\{q^b,q^s,y,m_{+1}\right\}} z\left[u(q^b) - c(q^s) + \lambda\left(y - \left(R^s + c\left(q^b\right)\right)/\Omega\right) + \pi\left(Y + \left(R^b - u(q^s)\right)/\Omega\right)\right] \\ &\qquad +\phi\left(m - y\right) + \beta V(m + \tau + z\left(u(q^s) - R^b\right)/\Omega - z\left(R^s + c\left(q^b\right)\right)/\Omega). \end{aligned} \quad (9)$$

The offers $\left(q^b, x^b\right)$ and $\left(q^s, x^s\right)$ satisfy the following conditions:

$$u'(q^b) = \frac{\lambda + \omega}{\Omega} c'(q^b) \quad (10)$$

$$c'(q^s) = \frac{\omega - \pi}{\Omega} u'(q^s) \quad (11)$$

$$\lambda\left(x^b - y\right) = 0 \quad (12)$$

$$\pi\left(x^s - Y\right) = 0 \quad (13)$$

Equations (10) and (11) are the first-order conditions with respect to q^b and q^s, and equations (12) and (13) are the Kuhn-Tucker conditions for the inequalities (5) and (6), respectively. Note that the first-best allocation requires $q = q^*$, where q^* satisfies $u'(q^*) = c'(q^*)$. This condition maximizes the total surplus in each match and the utility of the representative household. If the constraints (5) and (6) are not binding ($\lambda = \pi = 0$), the quantities produced and exchanges are efficient, i.e. $q^b = q^s = q^*$. If the constraints are binding ($\lambda, \pi > 0$), then $q^b, q^s < q^*$.[8]

[8] See the Appendix for a derivation of the terms of trade.

To determine the amount of money the household disburses to its members, differentiate (9) with respect to y to get

$$z(\lambda+\omega)\left(1-\frac{R_y^s}{\Omega}\right)-\phi-\omega z\leq 0 \quad (=0 \text{ if } y>0) \tag{14}$$

where R_y^s is the partial derivative of a seller's reservation utility with respect to the buyer's money holding y. Note that $R_y^b=0$ in (14) because a seller's constraint on money holdings is never binding, which implies that the partial derivative of a buyer's reservation utility is not affected when a seller of the representative household arrives with one additional unit of money in a match.

Finally, differentiating (9) with respect to m gives the following envelope condition:

$$\frac{\omega_{-1}}{\beta}=\phi+\omega \tag{15}$$

In a monetary equilibrium, $y>0$, substituting (14) into the envelope condition (15) gives

$$\frac{\omega_{-1}}{\beta}=z(\lambda+\omega)\left(1-\frac{R_y^s}{\Omega}\right)+(1-z)\omega. \tag{16}$$

Eliminating λ by its expression given in (10) and taking into account that in a symmetric equilibrium $\omega=\Omega$ and $q^b=q$, yields

$$\omega_{-1}=\beta\left\{z\omega\frac{u'(q)}{c'(q)}\left(1-\frac{R_y^s}{\omega}\right)+(1-z)\omega\right\} \tag{17}$$

The envelope condition (17) can be interpreted in the same way as an asset pricing equation. The left-hand side is the value in terms of utility of an additional unit of money at the end of the previous period. The right-hand side is the discounted value of holding this unit in the current period before the traders are matched. In the current period, with probability z a member is in a match where he can buy consumption goods. He acquires the goods if their consumption utility $\omega(1-R_y^s/\omega)u'(q)/c'(q)$ is larger than the indirect utility of hoarding the money ω. With probability $1-z$, a member has no opportunity to spend it, and consequently it is saved, which yields indirect utility ω. The term R_y^s/ω can be interpreted as a markup imposed by the seller.

By using $\gamma=m/m_{-1}$ (17) can be transformed to display the evolution of the real value of money holdings $m\omega$:

$$\frac{(m\omega)_{-1}}{m\omega}=\frac{\beta}{\gamma}\left\{z\frac{u'(q)}{c'(q)}\left(1-\frac{R_y^s}{\omega}\right)+(1-z)\right\} \tag{18}$$

In the following, we will focus our attention on equilibria where the real value of money holdings $m\omega$ is constant. In such a steady-state monetary equilibrium the envelope condition (18) can be written as follows:

$$\frac{\gamma-\beta}{z\beta}=\frac{u'(q)}{c'(q)}\left(1-\frac{R_y^s}{\omega}\right)-1 \tag{19}$$

Note that the envelope conditions in Berentsen and Rocheteau (2003), Lagos and Wright (2001), Rauch (2000), and Shi (1997, 1999, 2001) differ only in the specification of the holdup term R^s_y in (19). We will derive and discuss the role of the term R^s_y for the different models in Section 4.

Before we do so, however, let us discuss the role of money hoarding in this model. In the introduction, we claimed that if we allow for money hoarding by letting the households choose the amount of money their members carry to the search market, then steady-state monetary equilibria with $\gamma < \beta$ cannot exist. To see that this claim is true, note that in a monetary equilibrium, (14) holds with equality. Consequently, because the Lagrange multiplier ϕ is non-negative, in a symmetric steady-state monetary equilibrium (10) and (14) imply that

$$\frac{u'(q)}{c'(q)}\left(1 - \frac{R^s_y}{\omega}\right) \geq 1. \tag{20}$$

Consider (18) and assume that $\gamma < \beta$. Then, (20) implies that $(m\omega)_{-1}/m\omega > 1$, i.e. the real value of money converges to 0. Thus, if $\gamma < \beta$ there exists no symmetric steady-state monetary equilibrium.[9]

4 The hold-up term R^s_y

We first consider the case where households' trading strategies are not contingent on the money holdings of the potential opponents in bargaining. Thus, when they choose their money holdings households take the reservation values R^s of all other households as given. Consequently, $R^s_y = 0$ so that the envelope condition (19) reduces to

$$\frac{\gamma - \beta}{z\beta} = \frac{u'(q)}{c'(q)} - 1 \tag{21}$$

This would be the case, for example, if sellers could not observe buyers' money holdings.

To derive R^s_y when the households condition their trading strategies on the money holdings of their trading partners, note that from (3) R^s_y satisfies

$$R^s_y = (1 - \theta\Delta)\left[-c'(Q^s)\frac{\partial Q^s}{\partial y} + \Omega\frac{\partial X^s}{\partial y}\right] \tag{22}$$

From (22), when Δ goes to zero, the change in the seller's reservation value R^s_y is given by

$$R^s_y = -c'(Q^s)\frac{\partial Q^s}{\partial y} + \frac{\partial X^s}{\partial y}\Omega$$

[9]Finally, if $\gamma = \beta$, there exists a continuum of steady-state monetary equilibria with identical terms of trade satisfying $\left(1 - R^s_y/\omega\right) u'(q)/c'(q) = 1$. These equilibria only differ in their constant value of $m\omega$. Because there exists no steady-state equilibrium when $\gamma < \beta$ and a continuum of equilibria with money hoarding when $\gamma = \beta$, we assume for the rest of the paper that $\gamma > \beta$.

We will demonstrate later that in a monetary equilibrium the constraints on money holdings (5) and (6) are binding, which implies that $(Q^s, X^s) = (q(y), y)$.[10] Thus, if the constraint of the buyer (5) is binding

$$R_y^s = -c'(q)q'(y) + \omega \tag{23}$$

In a symmetric equilibrium where $\Omega = \omega$ and $Y = y$, the quantity of goods produced and exchanged $q(y)$ is implicitly defined by[11]

$$\frac{c'(q)}{u'(q)} = \frac{\theta\left[-c(q) + y\omega\right]}{(1-\theta)\left[u(q) - y\omega\right]} \tag{24}$$

Note from (24) that the surpluses of buyers and sellers satisfy

$$\begin{aligned} u(q) - y\omega &= \Theta(q)\left[u(q) - c(q)\right] && (25) \\ -c(q) + y\omega &= \left(1 - \Theta(q)\right)\left[u(q) - c(q)\right] && (26) \end{aligned}$$

where $\Theta(q) = \frac{\theta u'(q)}{\theta u'(q) + (1-\theta)c'(q)}$ is a buyer's share of the total match surplus. Note that $\Theta(q) > \theta$ if the trade is constrained by the buyer's money holdings. Thus, the division of the total surplus is determined by how severely the trade is constrained by the buyer's real money balance, and hence influenced by monetary policy. When money growth obeys the Friedman rule, the money constraint does not bind and so $\Theta(q) = \theta$.

Then from equation (25) we obtain

$$y\omega = g(q;\theta) \equiv \left[1 - \Theta(q)\right]u(q) + \Theta(q)c(q) \tag{27}$$

Total differentiating (27), we obtain[12]

$$q'(y) = \frac{\omega}{g'(q;\theta)} \tag{28}$$

Use (28) to eliminate $q'(y)$ in (23) to get

$$R_y^s = \frac{-c'(q)}{g'(q;\theta)}\omega + \omega \tag{29}$$

Finally, use (29) to eliminate R_y^s from (19) to obtain

$$\frac{\gamma - \beta}{z\beta} = \frac{u'(q)}{g'(q;\theta)} - 1 \tag{30}$$

[10] If they were not binding, then $(Q^s, X^s) = (q^*, x)$ where q^* is the efficient quantity and $x < y$. Consequently, $R_y^s = 0$. Note that R_y^s is not continuous at $q = q^*$. We therefore adopt the convention that R_y^s is equal to its left derivative at this point, that is, $R_y^s\big|_{q=q^*} = \lim_{q \to q^{*-}} R_y^s$.

[11] For details, see the Appendix.

[12] To compute this derivative, we use the assumption discussed above that in the bargaining the traders attribute to each other the economy-wide average value Ω for the indirect marginal utility of money. See Section 6 and the Appendix for a discussion for the case of an observable Ω.

where

$$g'(q;\theta) = \frac{(1-\theta)\theta\left\{c''(q)u'(q) - u''(q)c'(q)\right\}\left[u(q) - c(q)\right] + c'(q)u'(q)\left[\theta u'(q) + (1-\theta)c'(q)\right]}{\left[\theta u'(q) + (1-\theta)c'(q)\right]^2}.$$

Equation (30) equals the envelope condition in Lagos and Wright (2001). This result is of particular interest because our model differs in two respects from Lagos and Wright (2001). First, we use a different formalization devise to render the distribution of money holdings degenerate (the large household). Second, we consider a sequential bargaining game and not the axiomatic Nash bargaining solution as they do. Furthermore, for $\theta = \frac{1}{2}$ and $u(q) = q$ equation (30) replicates the envelope condition of Rauch (2000, eq. (25)), who also imposes the axiomatic symmetric Nash bargaining solution. Finally, note that when $\theta = 1$ the envelope condition (30) collapses to (21).

5 The Friedman rule

The Friedman rule requires deflating the nominal stock of money approximately at the rate of time preference. Because the utility of a representative household is $z\left[u(q) - c(q)\right]$, the Friedman rule is optimal in our model if the monetary policy that consists of setting $\gamma = \beta$ asymptotically maximizes the purchasing power of money by bringing q as close as possible to q^*.

In Shi (1997, 1999, 2001) and Berentsen and Rocheteau (2003) the envelope condition is given by (21); this implies that the Friedman rule holds for any value of θ because q approaches q^* as γ tends to β. Thus, in these models the Friedman rule attains the first-best allocation. The reason for this result is that a buyer can get the full return on his marginal unit of money, because of the inability of sellers to capture part of the surplus that an additional unit of money generates for the match.

In the following we consider the validity of the Friedman rule when the households condition their bargaining strategies on the money holdings of their trading partners as in Rauch (2000) and Lagos and Wright (2001). For this purpose, we use equation (30) to define a symmetric monetary steady-state equilibrium.

Definition 1 *For all $\gamma > \beta$, a symmetric monetary steady-state equilibrium is a $q > 0$ satisfying equation (30).*

For the following Proposition we assume that $\lim_{q\to 0} \frac{u'(q)}{g'(q;\theta)} = \infty$. This condition is a sufficient condition to guarantee the existence of a monetary equilibrium. It is satisfied, for example, for $u(q) = q^a$ and $c(q) = q^b$ with $0 < a < 1 < b$.

Proposition 1 *If $\theta = 1$, for all $\gamma > \beta$ a unique symmetric monetary steady-state equilibrium exists and $\lim_{\gamma\to\beta} q = q^*$. If $\theta \in (0,1)$, for all $\gamma > \beta$ there exists a symmetric monetary steady-state equilibrium such that $\lim_{\gamma\to\beta} q = \hat{q}(\theta) < q^*$, where $\hat{q}(\theta)$ is defined as the value of q that satisfies $u'(q) = g'(q;\theta)$.*

Proof. Define $\Psi(q;\theta) = \frac{u'(q)}{g'(q;\theta)}$. Assume first that $\theta = 1$. The envelope condition is identical to (21), i.e. $\Psi(q;1) = \frac{u'(q)}{c'(q)}$. The function $\Psi(q;1)$ is strictly decreasing, $\Psi(0;1) = +\infty$, and $\Psi(q^*;1) = 1$. Therefore, for all $\gamma > \beta$ there is a unique monetary equilibrium. Furthermore, q is a decreasing function of γ, and $\lim_{\gamma\to\beta} q = q^*$.

Assume next that $\theta < 1$. From the definition in the Proposition $\hat{q}(\theta)$ is the value of q such that $\Psi(q;\theta) = 1$. Note that $\hat{q}(\theta) < q^*$ for all $\theta < 1$. The function $\Psi(q;\theta)$ is continuous for all $q \in (0, \hat{q}(\theta)]$, $\Psi(0;\theta) = +\infty$, and $\Psi(\hat{q}(\theta);\theta) = 1$. Therefore, for all $\gamma > \beta$ there exists a $q \in (0, \hat{q}(\theta)]$ such that (30) holds. Finally, the continuity of $\Psi(.)$ implies that there exists a monetary equilibrium such that $\lim_{\gamma\to\beta} q = \hat{q}(\theta) < q^*$.

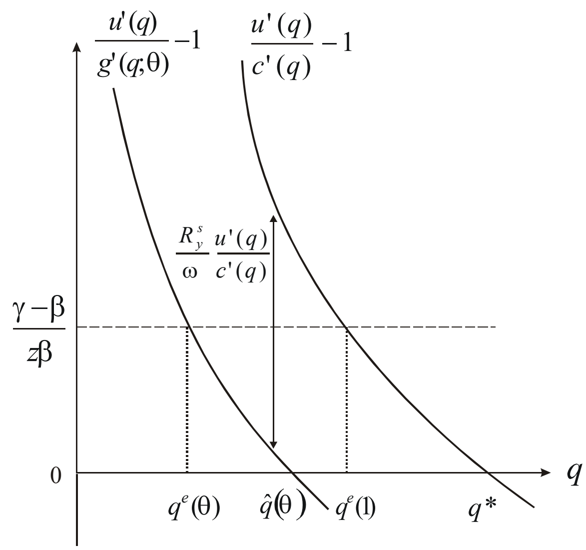

Figure 1. Determination of the equilibrium.

The determination of the equilibrium value q, denoted $q^e(\theta)$, is illustrated in Figure 1 above. The first curve represents the right-hand side of (30) when $\theta < 1$ and the second curve the right-hand side of (30) when $\theta = 1$. The quantity $\widehat{q}(\theta)$ is the quantity attained under the Friedman rule ($\gamma \to \beta$). Note that any equilibrium quantity $q^e(\theta) \leq \widehat{q}(\theta)$ can be replicated in the model without a hold-up problem, i.e. in Shi's approach, by simply increasing the gross growth rate γ appropriately as indicated by the term $\frac{R_y^s u'(q)}{\omega u'(q)}$.■

Proposition 1 confirms that the optimal monetary policy is the Friedman rule. This result is robust, because it is shared by all search models with divisible money, divisible goods, and a degenerate distribution of money holdings. Thus, in the absence of distributional effects such as those studied by Deviatov and Wallace (2001) and Molico (1997) and in the absence of search externalities as studied in Berentsen, Rocheteau, and Shi (2001), there is no welfare gain by choosing a rate of growth of the money supply larger than what is prescribed by the Friedman rule.

Nonetheless, according to Proposition 1, for $\theta < 1$, the Friedman rule fails to generate the efficient quantity of trade in each match. This inefficiency arises because at the time of production agents cannot contract with their future trading partners how much consumption goods they will receive in return for the acquired money. Accordingly, when buyers spend an additional unit of money, the cost of acquiring the unit is sunk, and consequently they are not able to appropriate the full return of a marginal unit of money, unless they have all the bargaining power. This inefficiency is due to a standard *holdup problem*, which arises in environments with bargaining and incomplete contracts. If buyers cannot get the full return of an additional unit of money, they reduce their initial investment, that is, they produce less to obtain the unit. This inefficiency cannot be corrected by the Friedman rule: The Friedman rule only corrects inefficiencies that are associated with the fact that agents discount future utilities.

Correction of the holdup inefficiency would require a higher rate of deflation than the Friedman rule, which is inconsistent with the existence of a steady-state monetary equilibrium when agents are allowed to hoard money. Such a policy would be only feasible if the households were constrained to bring either no money or all their money holdings into the matches. This explains why the first-best allocation can be attained in models with indivisible money, in which agents use lotteries to determine the terms of trade even though a holdup problem similar to the one emphasized in this paper exists (e.g. Berentsen, Molico, and Wright 2002). With indivisible money buyers cannot hoard money, i.e. they are technically forced to bring all their money holdings into a match. If we had constrained the household to redistribute all its money across members at the beginning of each period, i.e., $y = m$, then the optimal gross growth rate of the money supply γ^* would have been smaller than the discount factor β for all $\theta \in (0, 1)$ and would have guaranteed the first-best allocation.

6 Discussion

In this Section we first illustrate the determination of the terms of trade and the real value of money in Shi's approach and in the models of Lagos and Wright (2001) and Rauch (2000). We then discuss the formation of the terms of trade when the marginal value of money ω is observable. Finally, we conclude Section 6 with an interpretation of Shi's approach.

Determination of the terms of trade and the real value of money Figure 2 plots the buyer's surplus, $u(q) - y\omega$, as a function of the quantity traded, q, for the different pricing mechanisms investigated in the paper. First, if the buyer has all the bargaining power, $\theta = 1$, the buyer's surplus is equal to $u(q) - c(q)$. This function reaches a maximum at $q = q^*$. This can be seen from the curve labelled $u(q) - c(q)$ in Figure 2. Second, if the seller has some bargaining power and if the seller's reservation utility R^s is not made contingent on the buyer's money holdings, as in Shi's approach (1997), then the buyer's surplus is $u(q) - c(q) - R^s$. Again, this function attains its maximum at $q = q^*$. In both cases, the buyer has full bargaining power on his marginal unit of money which explains why his surplus is maximized at $q = q^*$. If the opportunity cost of holding money is zero, which is achieved through the Friedman rule, then the quantity traded will maximize the buyer's surplus, i.e. $q = q^*$. Third, if the seller's acceptance rule is made contingent on the level of money holdings of the buyer in bargaining, as in Lagos and Wright (2002) and Rauch (2000), then the surplus of the buyer is $u(q) - g(q;\theta)$. This function reaches a maximum at $q = \hat{q} < q^*$. It is labelled $u(q) - g(q;\theta)$ in Figure 2.

In order to see how the hold-up problem affects the terms of trade in an out-of-equilibrium match, assume that the same equilibrium quantity Q is traded in Shi's approach and in Lagos and Wright (2001) or Rauch (2000) (see Figure 2). Then, any differences in the two models arise in out-of-equilibrium matches only (when the buyer's money holdings is different from the equilibrium amount M). The different behavior in an out-of-equilibrium match is reflected in the different slopes of the functions $u(q) - c(q) - R^s$ and $u(q) - g(q, \theta)$ at $q = Q$ (see Figure 2). The curve that represents the buyer's surplus in Shi's framework is steeper than the curve that represents Lagos and Wright (2001) or Rauch (2000) because in Shi's framework the buyer gets the full return on his marginal unit of money.

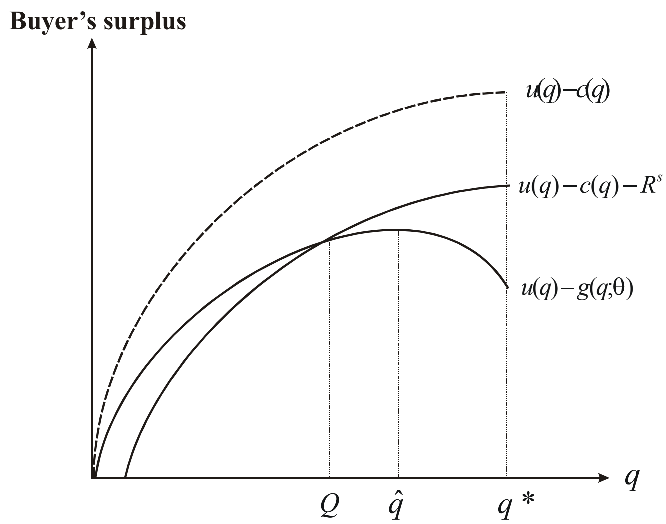

Figure 2. Determination of the terms of trade.

Finally, the fact that the buyer's surplus slops downward when $q \in (\hat{q}, q^*)$ reflects the presence of the hold-up problem. If $q > \widehat{q}$ and if a buyer brings an additional unit of money into a match, the increase of the seller's reservation value reduces the buyer's surplus. Consequently, the value of an additional unit of money in such a match is smaller than ω when $q > \widehat{q}$ so that household prefer to hoard money. Hence, no steady-state monetary equilibrium can exist with $q \in (\hat{q}, q^*)$.

Observable marginal value of money ω Throughout the paper we have assumed that in a match the marginal value of money ω is private information. If, instead, the marginal value of money ω is observable, the envelope condition (see Appendix 2 for the derivation) becomes

$$\frac{\gamma - \beta}{z\beta} = \frac{u'(q)}{g'(q;\theta)} + \frac{\varepsilon\,(1 - 2\theta)\,u'(q)\,c'(q)}{g'(q;\theta)\left[\theta u'(q) + (1-\theta)c'(q)\right]} - 1, \tag{31}$$

where $\varepsilon = \frac{d\omega}{dy}\frac{y}{\omega}$ is the elasticity of the marginal value of money with respect to y.

The novelty is the second term on the right-hand side of (31), which we refer to as the strategic effect. The strategic effect captures how the terms-of-trade are affected when the marginal value of money changes in an out-of-equilibrium match. While the hold-up problem (represented by the first term on the right-hand side of (31)) always depresses the value of money, the impact of the strategic effect depends on ε and θ. If the marginal value of money is decreasing in money holdings m and if $\theta < 1/2$, the strategic effect depresses the value of money even further. In contrast if $\theta > 1/2$, it has a positive impact on the equilibrium value of money. The problem is that we cannot say anything about the overall effect because we cannot derive the elasticity ε. Note however that with symmetric bargaining ($\theta = 1/2$) the elasticity ε vanishes in (31). Consequently, if $\theta = 1/2$, then the model generates the same envelope condition regardless of whether the marginal value of money is assumed to be observable or not. Finally, in Lagos and Wright (2001) there is no strategic effect because the marginal value of money ω is a market price which is taken as given by all agents.

Interpretation of the different approaches Shi's approach and the approach by Rauch (2000) have in common that they involve trading mechanisms that generate allocations in the pairwise meetings that are incentive-feasible and pairwise Pareto efficient. That is, both approaches select an allocation on the Pareto frontier of each bargaining set in each meeting. The main difference is that in Shi's approach the buyers have the full bargaining power on their marginal unit of money, which involves different bargaining weights in equilibrium and in out-of-equilibrium matches. To the extent that choosing an allocation for a bilateral match always involves some arbitrariness, one trading mechanism is not a priori more reasonable than another. Different trading mechanisms involve different protocols of bargaining (extensive forms) and the protocols might depend on the characteristics of the players, that is, there might be different protocols for equilibrium matches and out-of-equilibrium matches, where a player holds a different amount of money than what is expected in equilibrium.[13]

Finally, as mentioned before, in Lagos and Wright (2001) agents can trade money in a centralized market at some given market price ω. Lagos and Wright (2001) show that if the agents have quasi-linear preferences, then the distribution of money holdings in the search market will be degenerate. In the present paper we have seen that the closed form solutions in Shi's approach (in the version of Rauch with the hold-up problem) and in Lagos and Wright are identical. Consequently, the main difference between the approaches of Lagos and Wright and Shi is that in Shi the pooling of money holdings is within households whereas in Lagos and Wright the pooling of money holdings is among all agents in the economy, which implies that in Shi the marginal value of money ω is household specific whereas in Lagos and Wright it is a market price. The models have in common that the pooling generates degenerate distributions of money holdings, which renders tractable models of fully divisible money.

[13] Another paper that assumes that a player's bargaining weight is a function of the match type is Ravikumar and Wallace (2003).

7 Conclusion

This paper has studied the optimality of the Friedman-rule in search-theoretic models with divisible money, divisible goods, and a degenerate distribution of money holdings. The following results have emerged from our analysis. First, we have shown that the different formalization devices proposed by Shi (1997, 1999, 2001) and by Lagos and Wright (2001) are equivalent with respect to their closed-form solutions. Second, in the absence of search externalities and distributional issues, the Friedman rule is the optimal monetary policy. Third, we have explained the different results with respect to the efficiency of the monetary equilibrium under the Friedman rule reported in the search literature of divisible money. If, as in Shi's approach and in Berentsen and Rocheteau (2003), the households do not condition their trading strategies on the money holdings of their potential trading partners, the Friedman rule implements the first best. In contrast, if, as in Lagos and Wright (2001) and in Rauch (2000), the households condition their bargaining strategies on money holdings, then the Friedman rule does not guarantee the first best, unless buyers have all the bargaining power. The reason for this inefficiency is a holdup problem. A buyer who holds more money than what is expected in equilibrium cannot appropriate the entire surplus that this additional money provides to a match. In this sense, money is an asset whose holder — the buyer — is not able to capture its entire return, which results in an inefficiently low purchasing power of money even under the Friedman rule.

Appendix 1: Determination of the terms of trade Let us consider the bargaining solution in a single-coincidence meeting between a buyer of some household and a seller from some other household. In a symmetric equilibrium both agents hold Y units of money and the marginal value of money value for both households is Ω. Then, from (1), (2), (3) and (4) in a symmetric equilibrium (Q^b, X^b) and (Q^s, X^s) satisfy

$$\begin{aligned} -c(Q^b) + X^b\Omega &= (1-\theta\Delta)\left[-c(Q^s) + X^s\Omega\right], \\ u(Q^s) - X^s\Omega &= \left[1-(1-\theta)\Delta\right]\left[u(Q^b) - X^b\Omega\right] \end{aligned}$$

These two equations can be rewritten as

$$(1-\theta\Delta)c(Q^s) - c(Q^b) = \left[(1-\theta\Delta)X^s - X^b\right]\Omega, \tag{32}$$

$$u(Q^s) - \left[1-(1-\theta)\Delta\right]u(Q^b) = \left[X^s - \left[1-(1-\theta)\Delta\right]X^b\right]\Omega \tag{33}$$

From these equations, we deduce that as $\Delta \to 0$, Q^s and Q^b converge to the same value Q and that X^s and X^s converge to the same value X. Then, rewrite (32) and (33) to get

$$\left[-c(Q^b) + X^b\Omega\right] - \left[-c(Q^s) + X^s\Omega\right] = -\theta\Delta\left[-c(Q^s) + X^s\Omega\right], \tag{34}$$

$$\left[u(Q^s) - X^s\Omega\right] - \left[u(Q^b) - X^b\Omega\right] = -(1-\theta)\Delta\left[u(Q^b) - X^b\Omega\right] \tag{35}$$

Take the ratio of (32) and (33) and use the fact that $X^s = X^b = X$ to obtain

$$\frac{c(Q^s) - c(Q^b)}{u(Q^s) - u(Q^b)} = \frac{\theta\left[-c(Q^s) + X\Omega\right]}{(1-\theta)\left[u(Q^b) - X\Omega\right]} \tag{36}$$

Take the limit as Q^s and Q^b converge to Q to get

$$\frac{c'(Q)}{u'(Q)} = \frac{\theta\left[-c(Q) + X\Omega\right]}{(1-\theta)\left[u(Q) - X\Omega\right]} \tag{37}$$

We have to distinguish two cases. If the constraint (5) of the buyer's money holdings is binding, i.e. $\lambda > 0$, then $X = Y$ and $Q = q(Y) < q^*$, where q^* is the socially efficient value of q that satisfies $u'(q) = c'(q)$.[14] Note that if we replace Q by q and $Y\Omega$ by $y\omega$, then (37) is equal to (24).[15] If the constraint (5) is not binding ($\lambda = \pi = 0$), then $Q = q^*$, and $X \leq Y$, where from (37) X satisfies

$$X = \frac{(1-\theta)u(q^*) + \theta c(q^*)}{\Omega}.$$

Finally, note that terms of trade that we derived from a sequential bargaining procedure satisfy the Nash bargaining solution

$$\max_{Q,X} \left[u(Q) - X\Omega\right]^{\theta}\left[-c(Q) + X\Omega\right]^{1-\theta} \text{ s.t. } X \leq Y$$

[14] Note that if $X^b \leq Y$ binds then $X^s \leq Y$ must bind as well ($\pi > 0$). Suppose that this is not true. Then, $X^b = Y$ and $X^s < Y$. As Δ approaches 0 $Q^s > Q^b$. But from (11) and (12) $Q^b < q^* = Q^s$ which is a contradiction.

[15] The replacement takes into account that in equation (24) we consider a match between a buyer of household h who holds y units of money. Moreover, we consider a symmetric equilibrium where $\omega = \Omega$ and $q = Q$.

The benefit of the sequential bargaining procedure, in contrast to the axiomatic Nash bargaining solution, is that it reveals how the agents' behavior in out-of-equilibrium matches prevents the economy from attaining the first best under the Friedman rule.

Appendix 2. Observable marginal value of money To derive the envelope condition when ω is observable, we have to derive the derivatives $\frac{dq^b}{dy}$ and $\frac{dq^s}{dy}$, respectively. To derive $\frac{dq^b}{dy}$ we consider a match between a buyer of the representative household and a seller from some other household. Furthermore, we set $x^b = y$ because in the monetary equilibrium the constraint $x^b \leq y$ is binding. The terms of trade in such a match satisfy

$$\theta u'(q^b)\left[-c(q^b) + \Omega y\right] = (1-\theta)c'(q^b)\left[u(q^b) - \omega y\right]. \tag{38}$$

Totally differentiate (38), and note that in a symmetric equilibrium $Y = y$, $\Omega = \omega$, and $q^b = q$, to get

$$\frac{1}{\omega}\frac{dq^b}{dy} = \frac{1 + \varepsilon\left(1 - \Theta\left(q\right)\right)}{g'(q;\theta)} \tag{39}$$

where $\varepsilon = \frac{d\omega}{dy}\frac{y}{\omega}$ is the elasticity of the marginal value of money with respect to y. The derivative $g'(q;\theta)$ is defined in (30) and $\Theta\left(q\right) = \frac{\theta u'(q)}{\theta u'(q)+(1-\theta)c'(q)}$. We can distinguish between a hold-up effect and a strategic effect. The first term in the numerator represents the hold-up effect. It measure how the buyers consumption changes when y changes holding ω constant. The strategic effect is represented by the second term in the numerator and measure how the terms-of-trade are affected when the marginal value of money changes in an out-of-equilibrium match.

The tricky part is to derive ε and we are not able to do so. Note, however, that the strategic effect reduces the additional consumption that a buyer receives in a out-of-equilibrium match, and consequently will depress the equilibrium value of money if the marginal value of money is decreasing ($\varepsilon < 0$).[16]

To derive $\frac{dq^s}{dy}$ we consider a match between a seller of the representative household and a buyer from some other household. The terms of trade in such a match satisfy

$$\theta u'\left(q^s\right)\left[-c(q^s) + \omega Y\right] = (1-\theta)c'(q^s)\left[u(q^s) - \Omega Y\right]. \tag{40}$$

Totally differentiate (40) and then impose $Y = y$, $\Omega = \omega$, and $q^s = q$ to get

$$\frac{1}{\omega}\frac{dq^s}{dy} = \frac{\varepsilon\Theta\left(q\right)}{g'(q;\theta)} \tag{41}$$

Note that if the marginal value of money is decreasing ($\varepsilon < 0$), then $\frac{dq^s}{dy} < 0$. Thus, if $\varepsilon < 0$, the strategic effect reduces the quantity that a seller has to produce in a out-of-equilibrium match, and consequently will increase the equilibrium value of money.

[16] Models with a nondegenerate distribution of money holdings are characterized by a decreasing marginal value of money. See for example Berentsen (2002), Camera and Corbea (1999), Rocheteau (2000), and Zhou (1999).

In order to derive the envelope condition when the marginal value of money ω is observable, differentiate (9) with respect to y to get

$$z(\lambda+\omega)\left(1-\frac{R_y^s}{\Omega}\right)-z(\omega-\pi)\left(\frac{R_y^b}{\Omega}\right)-\phi-\omega z\leq 0 \quad (=0 \text{ if } y>0) \tag{42}$$

Note that in contrast to the first-order condition in the paper (14), the derivative R_y^b in (42) may be nonzero. This derivative takes into account that when a seller of the representative household arrives with one additional unit of money in a match, then the buyer may change its reservation utility because the marginal value of money of the seller's household may may be different from the equilibrium marginal value of money. Next use (10) and (11) to eliminate λ and π and assume that a monetary equilibrium exists ($y>0$) to get

$$z\frac{u'(q)}{c'(q)}\Omega\left(1-\frac{R_y^s}{\Omega}\right)-z\frac{c'(q)}{u'(q)}R_y^b-\phi-\omega z=0 \tag{43}$$

Use (43) to eliminate ϕ from the envelope condition (15) to get

$$\frac{\omega_{-1}}{\beta}=z\omega\frac{u'(q)}{c'(q)}\left(1-\frac{R_y^s}{\omega}\right)-z\frac{c'(q)}{u'(q)}R_y^b+(1-z)\omega$$

Then, in a symmetric equilibrium when $\Delta\to 0$ from (3) and (4) we have $R_y^s=\omega-c'(q)\frac{dq^b}{dy}$ and $R_y^b=u'(q)\frac{dq^s}{dy}$. Replace R_y^s and R_y^b to get

$$\frac{\omega_{-1}}{\beta}=z\left[u'(q)\frac{dq^b}{dy}-c'(q)\frac{dq^s}{dy}\right]+(1-z)\omega \tag{44}$$

In the steady-state the real value of money ωm is constant and we can express the envelope condition as follows

$$\frac{\gamma-\beta}{z\beta}=u'(q)\frac{1}{\omega}\frac{dq^b}{dy}-c'(q)\frac{1}{\omega}\frac{dq^s}{dy}-1 \tag{45}$$

Note that the left-hand side of (45) is equivalent to the left-hand side of the envelope condition we have used so far in this paper. In Shi's approach $\frac{dq^b}{dy}=\frac{1}{c'(q)}$ and $\frac{dq^s}{dy}=0$ so that (45) simplifies to

$$\frac{\gamma-\beta}{z\beta}=\frac{u'(q)}{c'(q)}-1 \tag{46}$$

Next use (39) and (41) to replace $\frac{dq^b}{dy}$ and $\frac{dq^s}{dy}$ in (45) to get (31).

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