Modeling Inconsistencies in People's Preferential Choices with Sequential Sampling Models

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Brief framework for the cumulative dissertation based on:

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Declaration

I, Nicolas Arnold Joel Berkowitsch (born 2nd October 1983 in Zurich, Switzerland), hereby declare the following:

(i) My cumulative dissertation is based on three manuscripts, of which one is in press one submitted, and one will be submitted soon. I contributed independently and substantially to all manuscripts in this dissertation and have been primarily responsible for the ideas, data collection, analyses, and writing of the papers. This characterization of my contributions is in agreement with my co-authors' views.

(ii) I only used the resources indicated.

(iii) I marked all the citations.

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Nicolas Berkowitsch

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Abstract

People are faced with hundreds of decisions every day, of which many involve choices between multi-attribute options. To study how people evaluate these options, different approaches have been suggested. One promising approach is to apply sequential sampling models (SSMs), which make predictions about the underlying cognitive processes involved in decision making. Multialternative Decision Field Theory (MDFT), a powerful SSM, assumes that people accumulate evidence for each option over time by continuously comparing the options' attribute values. A decision is made once the evidence of one option first passes a predefined threshold. Although MDFT has been widely used to predict people's choice behavior, an empirical model comparison is missing. This is mainly due to certain model restrictions. One such restriction is that the function to measure the distance between the options, is not fully specified. We addressed this challenge in the first manuscript and suggested a generalized distance function for multi-attribute choice options. In the second manuscript, we provided a testable version of MDFT and empirically compared the model to two of the most frequently used random utility models in the choice literature, the logit and probit models. We illustrated that when violations of consistency principles are likely to occur, MDFT described and predicted people's choice most accurately. In the third manuscript, we showed in an empirical experiment that previous choices can influence people's subsequent choice, even when independent sequential choices are stressed, suggesting another violation of consistency principles. Current SSMs, such as MDFT, cannot account for the observed compensating choice behavior, as shown in the manuscript. To overcome this limitation, we suggested a dependent sequential sampling model (DSSM), which assumes that the initial preferences for the choice options are updated after every choice. Using the preferential choice options of the experiment, we illustrated how DSSM can model the observed compensating choice behavior, as well as consistent choice behavior.

Introduction

Planning holidays involves many preferential choices. Where should I go? For an adventurous hike in the Himalayas? Or explore the unique underwater world of the Red Sea? Which airline should I take? With Swiss to earn miles for my frequent-flyer program or should I just book the cheapest easyJet flight? I should also seize this opportunity and finally buy a new digital camera to ensure everlasting memories of my trip. But which camera model? Nowadays, most of us make these purchases using online booking (e.g., www.ebookers.com/) or price comparison platforms (e.g., shopper.cnet.com/). To facilitate our task to make the “best” choice between the possible options, they are typically presented in a clear tabular arrangement. How would Mr. Holmes, a very principled man, plan his next holiday? Because Mr. Holmes adheres to very strict rules when faced with a decision, his preference for one option (e.g., flying with Swiss) over another option (flying with easyJet) would never be affected by the introduction of a new option (flying with Ryanair). Mr. Holmes also does not change his preferences after repeated choices, unless his previous choice caused grounds for complaint (e.g., repeated delay of the airline) or learned about an even better offer.

Those principles that Mr. Holmes is so proud of are part of a family of so-called consistency principles that are derived from logic or probability theory (Gigerenzer, 1996). Here, I focus on two principles: the independence from irrelevant alternatives (Luce, 1959) and independent sequential choices (e.g., Schwarz, 2011; for an overview of further consistency principles and a classification see Rieskamp, Busemeyer, & Mellers, 2006). According to the former, adding a new option to a choice set of two options should not change the ratio of the choice shares of the two options (Luce, 1959). The latter assumes that previous choices should not influence subsequent choices in the absence of learning or in situations without any feedback. Choice literature provides many examples of people violating these consistency principles (e.g., Ayton & Fischer, 2004; Clotfelter & Cook, 1993;

Huber, Payne, & Puto, 1982; Huber & Puto, 1983; Simonson & Tversky, 1992; Slovic & Tversky, 1974; Tversky 1972a, 1972b). Applied to the example above, those people would switch from Swiss to easyJet, after discovering a new offer from Ryanair providing the same (low) service as easyJet but at a higher price. On other occasions, they would feel licensed to switch from flying economy class to business class, after repeatedly flying economy class.

The goal of this cumulative dissertation is to better understand why (i.e., to explain) and when (i.e., to predict) these violations occur and how sequential sampling models (discussed in detail below) can be used to account for people's inconsistent preferences. I approach these questions by first reviewing selected papers in choice literature which report violations of the two consistency principles. Next, I will introduce the underlying idea of cognitive models and highlight their advantages compared to theories. In particular, I will discuss the application of *sequential sampling models* (SSMs) to model people's preferences. Following that, I will address the need to account for the preferential relationship between choice options (i.e., whether one option dominates the other or not), which will be the focus of the first manuscript. In this first manuscript, we suggested a generalized distance function for preferential choice options. Subsequently, in the second manuscript we incorporated this distance function into a powerful SSM and empirically tested it against standard random utility models in consumer choice experiments. The choice sets were designed in a way to increase the likelihood of observing violations of consistency principles (explained in more detail below). Finally, in the third manuscript, we used a job selection experiment to illustrate that preferential choices are highly influenced by previous choices, even when independent choices are stressed. We suggested how to extend current SSMs to incorporate the influence of previous choice as a function of changing initial preferences for the choice options.

Violations of Consistency Principles

Just over 60 years ago, Mosteller and Nogee (1951) observed that people's preferences are not necessarily consistent. When people were faced with monetary gambles with increasing rewards, the probability of choosing a risky gamble over a fixed amount gradually increased, resembling an S-shaped function. This contradicts expected utility theory, which predicts a sudden jump in probability in an all-or-nothing manner (von Neumann & Morgenstern, 1947). Even when faced repeatedly with the same sets of gamble pairs, people change their choices from one set to the other set in up to 23% of the cases (Hey, 2001). People's preferences are often not only inconsistent, but are also strongly influenced by previous choices. Schwarz (2011) analyzed the referee's penalty decisions in over 12,902 matches. Of particular interest were the 441 matches with exactly two penalties. If these penalty decisions were to be independent sequential decisions—which they should—it should not matter for the second penalty decision to which team the first penalty has been awarded. However, Schwarz (2011) found that teams awarded with the first penalty were less likely to be awarded with the second penalty and vice versa. Similar results have been found for foul calls in basketball (Anderson & Pierce, 2009). Such compensatory or balancing choices behavior has also been reported in the consumer and the moral context. Only people who had to imagine to have volunteered doing community service, were more likely to subsequently choose the more luxury options out of two consumer products (Khan & Dhar, 2006, Study 1). In a different Study, Monin & Miller (2001) found that people who could show to be free of prejudices (i.e., by disagreeing with sexist statements) were subsequently more likely to indicate that a male candidate was better qualified for a job opening than a female candidate.

Besides these violations of independent sequential choices, another violation of consistency is frequently observed after introducing a new choice option to a set of two comparable choice options. For example, introducing a new job candidate only increased the choice share of the existing candidate which dominated the new candidate (Highhouse, 1996). Sheng, Parker, and Nakamoto (2005) reported a similar violation of the independence from irrelevant alternative principle for choices between consumer products described on two quality attributes (e.g., battery life and sound quality). The choice shares of two options changed systematically in favor of the compromise option, after an extreme option (i.e., very high values one attribute and very low values on the other attribute) was introduced. A promising group of choice models that allows studying the nature of people's inconsistent preferences, violating consistency principles, are so-called *cognitive models*.

Cognitive Models

Cognitive models describe and explain the cognitive process underlying decisions, and how these processes interact with each other (Busemeyer & Diederich, 2010). They differ from theories that describe decision processes only verbally, in that cognitive models use a formal mathematical language for these processes (Busemeyer & Diederich, 2010). The mathematical assumptions underlying cognitive models are typically based on principles of cognition, such as decay in memory or attention-switching processes (e.g., Roe, Busemeyer, & Johnson, 2001). SSMs are a powerful group of cognitive models which assume that people accumulate preferences for each available option over time (e.g., Busemeyer & Townsend, 1993; Ratcliff, 1978; Townsend & Ashby, 1983; Vickers, 1970; Usher & McClelland, 2004). Depending on the stopping rule applied, either the option that is first to reach a predefined threshold (internal stopping rule) or the option with the highest accumulated preference after a fixed deliberation time (external stopping-rule) is chosen. The mathematical formalization of SSMs provides many advantages over theories: Unlike theories, the predictions of SSMs can be distinguished even when their qualitative predictions do not differ (i.e., both predict option A to be chosen). Further, SSMs produce logically valid predictions (Busemeyer & Diederich, 2010). Because they "incorporate the time course of events" (Johnson, 2006, p. 635), they predict how the preferences for different choice options change as a function of time, and thus allow reaction time predictions. Finally, using formalized models can also help us to describe complex behavior that could not be easily described verbally (Lewandowsky & Farrell, 2011).

SSMs have become increasingly popular in the past and have been successfully applied in many fields: In clinical research to identify different decision processes between clinical and nonclinical individuals (White, Ratcliff, Vasey, & McKoon, 2010), in neuroscience to understand value-based decisions and their underlying cognitive and neural mechanism (Gluth, Rieskamp, & Büchel, 2012), in machine learning to improve the interaction between humans and machines (Gao & Lee, 2006; Lee, Son, & Jin, 2008), in categorization to predict the perceptual classification of objects (Nosofsky & Palmeri, 1997), in memory research to test the accuracy of memory retrieval (Ratcliff, 1978), in risky decision making to predict choices between monetary gambles (Busemeyer & Townsend, 1993), in sports to model individual differences in risk-taking behavior (Raab & Johnson, 2004), and to study decision making under time pressure (Diederich 2003; Dror, Busemeyer & Basola, 1999).

Multialternative Decision Field Theory

One promising SSM is *Multialternative Decision Field Theory* (MDFT; Roe et al., 2001). MDFT has been used to predict people's inferential choices (Trueblood, 2012), perceptual choices (Trueblood, Brown, Heathcote, & Busemeyer, 2013), risky choices in sports decisions (Johnson, 2006), and preferential choices (Roe et al., 2001), just to name a few. Despite MDFT's popularity, an empirical comparison to other choice models is missing. In previous model comparisons researchers have resorted to theoretical comparisons (e.g., Busemeyer, Barkan, Mehta, & Chaturvedi, 2007; Pettibone, 2012; Rieskamp et al., 2006), to fixed sets of parameters (e.g., Johnson, 2006) or applied time-intensive parameter simulations (e.g., Trueblood, Brown, & Heathcote, 2013; Tsetsos, Usher, & Chater, 2010). However, predicting people's choices from a given set of parameters is not the same as actually estimating the model's parameters from empirical data. Thus, it is an open question as to how well MDFT performs compared to other choice models with fewer parameters. This is because certain model restrictions need to be overcome (Otter et al., 2008). One of the restrictions is that a generalized distance function to describe preferential choice options in the multi-attribute space is missing and has yet to be specified. In the first manuscript we addressed this issue and provided such a generalized distance function, whereas in the second manuscript we empirically tested MDFT against two prominent choice models, the logit and the probit models. Finally, in the third manuscript we showed how in principle independent consecutive choices influence the subsequent choice. Using a job selection task we experimentally illustrated how SSMs can be extended to incorporate the influence of previous choices on subsequent choices.

Manuscript One: A Generalized Distance Function for Preferential Choices

Berkowitsch, N. A. J., Scheibehenne, B., Rieskamp, J., & M. Matthäus. A Generalized Distance Function for Preferential Choices. Manuscript submitted for publication.

When faced with a decision between multiple options, people with consistent preferences like Mr. Holmes evaluate these options independently. However, past work provides many examples of people who—unlike Mr. Holmes—evaluate choice options relative to each other (e.g., Huber et al., 1982; Rieskamp, et al., 2006; Simonson & Tversky, 1992; Slovic & Tversky, 1974). Thus, to accurately predict people's preferential choices between options, choice models need to account for interdependent evaluation. One approach suggests that people evaluate options according to their subjective similarity in the multi-attribute space (Roe et al., 2001; Rooderkerk, Van Heerde, & Bijmolt, 2011). Similar to the concept of lateral inhibition (McClelland & Rumelhart, 1981), this approach assumes that more similar (i.e., closer) options compete more strongly with each other than dissimilar (i.e., more distant) options, where similarity is defined as a decreasing function of the distance between the options (Nosofsky, 1984; Shepard 1987).

Previous distance functions, such as the Euclidian distance function, simply calculate the objective distance between the options in the multi-attribute space, ignoring the importance that people assign to the attributes. In fact, multiple studies in perceptual categorization revealed that the objective distance between two objects can strongly differ from their psychological distance (e.g., Nosofsky & Johansen, 2000). Nosofsky (1986) suggested applying a weighted Minkowsky metric that shrinks or stretches the multi-attribute space proportional to the attention that an individual assigns to each dimension (Nosofsky, 1986). Whereas this approach accounts for individual differences, both the Euclidian distance and the weighted Minkowsky metric ignore the preferential relationship between the options. That is, they cannot distinguish whether one option *dominates* the other option (dominant pairs), and hence is preferred, or whether an individual is *indifferent* between both options (indifferent pairs), because they are equally attractive. Huber et al. (1982) suggested distinguishing the distance between dominant and indifferent pairs and weighting distance in dominance direction more strongly than distance in indifference direction. Hotaling, Busemeyer, & Li (2010) formalized this idea, but their distance function is limited to choice options described on two attributes with equal attribute weights. However, many decisions involve choice options described on multiple attributes with unequal attribute weights. In sum, previous distance functions either ignore the preferential relationship between the options or cannot account for individual attribute weights.

Towards a more general approach to measuring the distance between preferential choice options, we developed a distance function that simultaneously overcomes these limitations. It distinguishes the preferential relationship between the options (i.e., whether one option is dominant or not), is flexible enough to allow individual attribute weights, and can calculate the distance between preferential choice options described on multiple attributes. Similar to previous approaches, we distinguished between distance in dominance direction, captured by a dominance vector, and distance in indifference direction, captured by an indifference vector (Hotaling et al., 2012; Huber et al., 1982). We defined the indifference vector to describe the exchange ratio between two attributes; that is, the exchange ratio expresses how many units of one attribute an individual is willing to give up to increase the other attribute by one unit. This allows accounting for individual differences assigned to the attribute weights and results in more or less steep indifference vectors. The dominance vector is defined to be orthogonal to the indifference vector (Tversky, Sattah, & Slovic, 1988; Wedell, 1991). Because for options with more than two attributes multiple exchange ratios exist, multiple indifference vectors are required to define the distance in indifference direction. In contrast, a single dominance vector is sufficient (i.e., an option either dominates another option or it does not). The number of indifference vectors necessary to describe all possible exchange ratios is one smaller than the number of attributes describing the choice options. Accordingly, the dominance vector needs to be orthogonal to all indifference vectors. As suggested by Huber et al. (1982), we included an individual weighting parameter that places higher weight on distance in dominance direction relative to distance in indifference direction. Following the mathematical derivation, we provided a hands-on example to compare our generalized distance function to previous distance measures. The example illustrated that when the above-described requirements for a generalized distance function are only partially fulfilled, incorrect conclusions about the psychological distance, and thus the perceived similarity between options, are drawn.

Finally, we suggested how future work could test whether the assumptions we made to define the generalized distance function yield a more accurate description of people's perceived similarity. We assumed linear exchange ratios when defining the indifference vectors, implying that a lower value of one attribute can always be compensated by an increase in the value of another attribute. Although this implementation is straightforward, this assumption might sometimes be violated. For example, we might require a minimum legroom space in the airplanes and are not willing to go below that, even when compensated with a lower price. On the other hand, increasing the legroom space by one unit might only be balanced by a ticket price increase outside of our budget constraint. Thus, future work could test whether a linear indifference approach or a convex indifference approach provides a better account for the perceived similarity between preferential choice options.

To conclude, the generalized distance function provides a feasible measure to describe the distance between preferential choice options in the multi-attribute space and accounts for individual differences. These properties allow us to incorporate this distance function into choice models (such as SSMs) that estimate the subjective importance people assign to the different attributes and to increase their predictive accuracies (e.g., Roe et al., 2001; Rooderkerk et al., 2011).

Remember, one of the reasons why MDFT has never been empirically tested against other choice models is that its distance function was not fully specified (see above). Now that we have overcome this hurdle, we can focus on the next challenge to make MDFT testable, namely to provide an analytical solution to directly calculate the choice probabilities, thereby avoiding time-intensive parameter simulations. We addressed this challenge in the second manuscript, which follows next.

Manuscript Two: Testing Multialternative Decision Field Theory

Berkowitsch, N. A. J., Scheibehenne, B., & Rieskamp, J. (in press). Rigorously testing Multialternative Decision Field Theory against standard random utility models. *Journal of Experimental Psychology: General*.

SSMs, which aim for a better explanation of human decision making, have become increasingly popular. They describe the underlying processes of decision making, which are typically based on principles of cognition (e.g., Lewandoswky & Farrell, 2011; Busemeyer & Diederich, 2010). These models assume that people accumulate preferences for each available option before making a decision. The preference of the option first to pass a predefined threshold or the option with the highest preference after a fixed deliberation time is assumed to be the chosen option.

One promising and powerful SSM is MDFT (Roe et al., 2001). MDFT assumes that the accumulation process of the options' preferences is a function of the importance weights that people assign to the attributes describing the options (e.g., price of the flights, legroom of the seats in centimeters), an attention-switching process between these attributes, a decay process, and an inhibition process, where closer (i.e., more similar) options are assumed to inhibit each other more strongly than distant (i.e., less similar) options (for details see Roe et al., 2001). The model captures these processes with several free parameters. However, a higher number of parameters inherently increases the flexibility of a model, and thereby its complexity.

On the other hand, random utility models (RUMs), focusing solely on the decision outcome, have largely dominated the field of decision research. RUMs build on expected utility theory that assumes that the option with highest utility is chosen (von Neumann & Morgenstern, 1947). By including an error term to account for the randomness in people's choices, RUMs allow probabilistic choice predictions (for an overview see McFadden, 2001; Train, 2001). Two of the most prominent and widely used RUMs are the logit and probit models (e.g., Daganzo, 1980; Hausman & Wiese, 1978; Luce, 1959; McFadden, 1973; Thurstone, 1927). They enjoy great popularity across many research domains, such as political decision making (e.g., Bowler, Karp, & Donovan, 2010), travel behavior (e.g., Adamowicz, Louviere, & Williams, 1994), food choices (e.g., Green & Srinivasan, 1990), or consumer choices (e.g., Loureiro & Umberger, 2007). Presumably, their popularity is largely a result of their ease of implementation and the ease of estimating the models' parameters.

In contrast, estimating the model parameters of MDFT is not straightforward and requires time-intensive parameter simulations (e.g., Trueblood et al., 2013). This is because an analytical solution to directly calculate the choice probabilities MDFT is missing. Another challenge is to define an adequate distance function for options in the multi-attribute space with unequal attribute weights. Once these challenges are overcome, we can conduct an empirical model comparison between MDFT and RUMs, such as the logit and probit models. This comparison of a process-driven approach against an outcome-focused approach has often been called for in the choice literature (e.g., Chandukala, Kim, Otter, Rossi, & Allenby, 2007; Otter et al. 2008; Reutskaja, Nagel, Camerer, & Rangel, 2011) and provides a valuable insight into whether the higher model complexity of MDFT compared to the logit and probit models is justified by a higher model fit.

In two studies we tested MDFT rigorously against the logit and probit models by different means of model comparison. To overcome the challenges described above, we first incorporated the generalized distance function (Berkowitsch, Scheibehenne, Rieskamp, & Matthäus, 2013) into MDFT and, second, assumed infinite decision time, which allows direct calculation of choice probabilities (see Appendix in Berkowitsch, Scheibehenne, & Rieskamp, 2013 for details on the mathematical derivations). In each of the two experiments in Study 1, 30 participants repeatedly chose their favorite digital camera out of digital camera triplets, described on multiple attributes. The first experiment served as a calibration experiment to compare the models on a descriptive level, whereas the second experiment served as a generalization experiment to compare the models on a predictive level (Busemeyer & Wang, 2000).

The model comparison in the calibration experiment revealed that MDFT described participants' choices more accurately than the RUMs or a baseline model, assuming random choices. However, when the model complexity was taken into account using the Bayesian Information Criterion (Raftery, 1995), MDFT's advantage over the RUMs largely disappeared and the logit model, followed by the probit model were classified as the most likely models to have generated most of the observed data. For the generalization experiment, the probit model and MDFT were the most accurate models for predicting participants' choices.

In Study 2 we intended to create choice sets in which participants would violate the independence from irrelevant alternatives principle, according to which adding a new option to a choice set of two options, should not change the ratio of the choice shares of the two options (Luce, 1959). Because the logit and the probit models adhere to this principle, MDFT should outperform the logit and probit models. Three well-researched context effects known to violate the independence from irrelevant alternatives principle are the attraction, compromise, and similarity effect. The *attraction effect* predicts that adding an option dominated by one of two existing options increases the choice share of the dominating option (Huber et al., 1982; Huber & Puto, 1983). According to the *compromise effect*, adding an extreme option increases the relative choice share of the existing option now becoming the compromise option (Simonson & Tversky, 1992; Tversky & Simonson 1993). Finally, the *similarity effect* refers to choice situations where adding a similar option to two existing options hurts the choice share of the more similar option and thereby increases the relative choice share of the dissimilar option (Tversky, 1972a; 1972b).

We calculated the individually added choice option using a matching task (Carmon & Simonson, 1988). Pairs of consumer options described on two attributes (e.g., "price" and "quality") were repeatedly presented to participants, while the attribute value of one option was missing. We asked participants to fill in the missing value so that the two options became equally attractive. The matched option predicted to be chosen more frequently is referred to as the *target*, whereas the remaining matched option is referred to as the target's *competitor*. For each matched choice pair we calculated one new choice option, resulting in either an attraction, a compromise, or a similarity choice triplet. Participants were presented with these individually created choice triplets in a randomized order a couple of days later and were repeatedly asked to choose their favorite product. The total numbers of attraction, compromise, and similarity choice triplets were evenly balanced within participants.

We tested whether systematically adding the new choice options resulted in an increase of the choice share of the target relative to its competitor. Thereto, we calculated for each of the three context effects the 95% highest posterior density interval representing the most credible values of the relative choice share of the target using Bayesian statistics (Kruschke, 2011a, 2011b). The results indicated reliable attraction and compromise effects, and a weak similarity effect, suggesting frequent violations of the independence from irrelevant alternatives principle. Interestingly, people who were prone to either the attraction or the compromise effect were unlikely to show the similarity effect. On the other hand, people who were prone to the similarity effect were unlikely to show the attraction or compromise effect. The model comparison revealed that MDFT was the most accurate model in *describing* participants' choices even when the model complexity was taken into account (Raftery, 1995). Supporting this result, participants' choices were also best *predicted* by MDFT compared to the RUMs, using cross-validation (Browne, 2000; Stone, 1974).

In the present manuscript we focused on testing MDFT, a promising SSM, against two of the most frequently applied RUMs in choice literature, the logit and probit models. So far, an empirical model comparison had been missing (e.g., Busemeyer et al., 2007; Pettibone, 2012; Rieskamp et al., 2006; Trueblood, Brown, & Heathcote, 2013; Tsetsos, et al., 2010). Here, we incorporated the generalized distance function of Berkowitsch et al. (2013) into MDFT and suggested infinite deliberation time. This allowed us to provide an empirically testable version of MDFT, as successfully confirmed across the two studies. The first study illustrated that for the context on hand, MDFT did not describe people's choices more accurately than the RUMs when complexity was taken into account. However, MDFT was comparable to the probit model and outperformed the logit model in predicting people's choices. When people were likely to evaluate the choice options interdependently, as was the case for Study 2, MDFT was best in describing and predicting the observed data. These findings imply that the application of SSMs, such as MDFT, is most advantageous when context effects are likely to occur.

In the third manuscript, we addressed a current limitation of SSMs. Whereas SSMs such as MDFT can account for interdependent evaluation of the choice options, they assume independent consecutive choices, as long as the choice environment does not provide any explicit feedback (e.g., learning). Using a job selection task we showed that this assumption is systematically violated and suggested how SSMs can be extended to take influence from previous choice into account to predict people's subsequent preferential choices.

Manuscript Three: Modeling Sequence Effects Using a Sequential Sampling Model

Berkowitsch, N. A. J., Scheibehenne, B., & Rieskamp, J. How Previous Choices Affect Present Choices: Explaining Choice Dependencies with a Sequential Sampling Model. Manuscript to be submitted for publication.

Many of our daily decisions involve sequential choices. For example, when going to a restaurant we choose between appetizers, main courses, and—if our budget and conscience allow—between different desserts. Most likely, our ultimate selection of the courses does not reflect isolated, but dependent choices. Cognitive models that aim to explain the underlying cognitive process of decision making can account for sequence effects by integrating a (reinforcement) learning parameter (Kruschke, 1992; Gluck & Bower, 1988; Rieskamp & Otto, 2006; Simão & Todd, 2002; Stewart, Brown, & Chater, 2002; Stewart & Brown, 2004; Todd, 2007). However, a large body of literature, especially in moral research, reports many cases where previous choices systematically influence subsequent choices, even when the choice environment does not provide any explicit feedback (e.g., learning). For example, only people who were previously given the opportunity to a condemn sexist or racist statement were subsequently more likely to prefer a male over female job candidate or a White over a Black job candidate (Effron, Miller, & Monin, 2012; Monin & Miller, 2001). This so-called *compensating choice behavior* and its counterpart, *consistent choice behavior*, violate the independent sequential choices principle and have been observed across many studies in moral research (e.g., Conway & Peetz, 2012; Gneezy, Imas, Nelson, Brown, & Norton, 2012; Jordan, Mullen, & Murnighan, 2011; Sachdeva, Illiev, & Medin, 2009; Zhang, Cornwell, & Higgins, 2013). Apart from a few exceptions (e.g., Zhang, et al., 2013), these sequence effects were only reported across different experimental tasks (e.g., a rating task following a choice task, different consecutive choice tasks). Thus, the current manuscript had two goals: First, we wanted to test whether such sequence effects were likely to occur in a repeated choice task. Second, if so, how do we need to extend current SSMs to account for these sequence effects?

Toward the first goal we invited participants to a job selection experiment, where they were repeatedly faced with the choice between pairs of job candidates. In the first eight of the ten pairs one candidate clearly dominated the other candidate (i.e., dominating pairs), whereas the job candidates in the remaining two pairs were equally qualified (i.e., non-dominating pairs). The job candidates were described on multiple attributes (e.g., leadership skills, social skills) and their gender was indicated by a profile picture. We varied the gender of the first eight dominating pairs across the four conditions, resulting either in a female-dominates-male condition, a male-dominates-female condition, a female-dominates-female condition, or a male-dominates-male condition. For the remaining two non-dominating pairs, the two job candidates always consisted of a female and a male candidate. The attribute values for all ten pairs across the four conditions were the same. Participants were randomly assigned to one of the four conditions and repeatedly had to choose their favorite job candidate for different companies, thus stressing independent choices. We randomized the order within the dominating pairs and within the non-dominating pairs.

The results indicated a strong influence of the previous choices between the dominating-pairs on the subsequent choice between the non-dominating pairs. Consistent with previous findings, we observed the highest choice share of the male candidate, when participants could show to be free from sexist prejudices (i.e., choosing the female candidate in the female-dominates-male condition). This suggested that sequence effects also play a major role in repeated choice tasks.

To account for sequence effects violating the independent repeated choice principle, we proposed a dependent sequential sampling model (DSSM) that builds on current SSMs. DSSM assumes three cognitive processes: First, choices made with little confidence are more likely to result in compensating choice behaviors, whereas choices made with strong confidence are more likely to result in consistent choice behavior. Second, more recent choices compared to more remote choices have a stronger influence on the subsequent choice. Third, whether compensating or consistent choice behavior results from previous choices varies—besides the level of confidence—across individuals and thus must be accounted for on an individual level.

DSSM assumes that sequentially choosing between similar choice pairs changes people's experience with the available options. SSMs account for previous experience by estimating the options' initial preferences. In contrast to SSMs, DSSM does not set the initial preferences to constant values over all choices, but assumes that previous choices update the subsequent initial preferences according to the three processes described above. In the manuscript we explained how these processes interact, as well as how we derived the equations to calculate the initial preferences for the subsequent decision. Using the attribute values of job candidates presented in the experiment, we compared the predictions of an SSM, such as MDFT, to DSSM for a given set of parameters. The model comparison successfully illustrated that DSSM can account for the observed compensating behavior, whereas MDFT cannot.

In this manuscript we provided important empirical insights into how previous choices can influence subsequent choices, even when the choice environment does not provide explicit feedback, such as learning. Current versions of SSMs cannot account for this empirical finding. Therefore, we suggested a DSSM that accounts for the influence of people's previous choices on the subsequent decision, resulting in either compensating or consistent choice behavior. Theoretically, DSSM can also account for other well-known sequence effects, such as the gambler's fallacy (Clotfelter & Cook, 1993) or the hot hand fallacy (e.g., Scheibehenne, Wilke, & Todd, 2011). As this advantage involves additional free parameters, future work should focus on an empirical comparison of DSSM to less complex choice models.

General Discussion

In this cumulative dissertation I aimed to promote the feasibility of SSMs to explain and predict human choice behavior. Toward this goal we illustrated how existing cognitive mechanisms describing the distance between choice options can be generalized, such that their application is not limited to specific choice environments. Further, we proposed a testable version of MDFT, a powerful SSM, and compared its accuracy to describe and predict people's choice behavior to two frequently used RUMs. Finally, we suggested how SSMs can be extended to account for the influence of previous choices when the choice environment does not provide any explicit feedback (e.g., learning).

Throughout the manuscripts we observed different violations of consistency principles: Introducing a new choice option to an existing set of choice options changed people's relative choice share of the existing options. Further, we observed compensating choice behavior in a choice environment not providing any explicit feedback, suggesting a strong influence of previous choices on the subsequent choice. But is consistent choice behavior something worth pursuing? Should we nudge people to consistent choice behavior

(Thaler & Sunstein, 2008)? On this note Gigerenzer (1996) argued that inconsistent choice behavior is not to be put on a level with irrational choice behavior. Whereas inconsistent behavior of machines might be fatal (e.g., malfunctioning traffic lights, defective flight traffic control systems), this is not necessarily the case for humans. Rather, inconsistency allows us to break free from rigid choice patterns and to adapt to the (changing) choice environment. In this sense, I strongly vote to focus our research effort in further developing cognitive models, such as SSMs, to better describe, explain, and predict people's choice behavior, instead of educating people how to make consistent choices.

To further develop SSMs, future work should pursue two approaches. On the one hand, scholars should continue challenging previous and improving current (cognitive) choice models by means of model comparisons. For example, it would be interesting to estimate MDFT's parameters using Bayesian techniques (e.g., Kruschke, Aguinis, & Joo, 2012). This allows researchers to study the estimated distribution of the parameters instead of point estimates, where more narrowly distributed parameters indicate higher certainty of the estimate (Kruschke, 2011a). Another approach to study the validity of SSMs is to investigate the psychological meaning of their parameters and of their cognitive measures. For example, to test the generalized distance function future work could present people with pairs of dominant- and non-dominant preferential choice options. But instead of choosing between the options, people should be asked to judge the perceived distance. This would provide empirical evidence as to whether people perceive distance in dominance direction as more distant relative to distance in indifference direction. Further, the decay parameter in MDFT reflects how quickly the preference state decays during deliberation. Researchers could correlate the decay parameter with people's working memory capacity (Unsworth, Heitz, Schrock, & Engle, 2005). Finally, in DSSM positive values of the compensating-consistency parameter reflect compensating choice behavior, whereas values less than or equal to zero reflect consistent choice behavior. Here, future work could test whether the sign and the magnitude of the parameter correlate with people's need for consistency (Guadagno & Cialdini, 2010).

To conclude, the novel contributions of this cumulative dissertation can be divided into three groups. First, we generalized previous (Manuscript 1) and *suggested* new *cognitive mechanisms* (Manuscript 3) that can be incorporated into choice models, such as SSMs. Second, we illustrated and discussed choice situations in which *empirically observed violations of consistency principles* are likely to occur (Manuscripts 2 and 3). Third, we provided new empirical *model comparisons* between an SSM and RUMs (Manuscript 2) and within SSMs (Manuscript 3).

References

Adamowicz, W., Louviere, J., & Williams, M. (1994). Combining revealed and stated preference methods for valuing environmental amenities. *Journal of Environmental Economics and Management*, *26*, 271–292.

Anderson, K. J. & Pierce, D. A. (2009). Officiating bias: The effect of foul differential on foul calls in NCAA basketball. *Journal of Sports Sciences, 27*, 687–694. doi: 10.1080/02640410902729733

Ayton, P., & Fischer, I. (2004). The hot hand fallacy and the gambler's fallacy: Two faces of subjective randomness? *Memory and Cognition*, *32*, 1369–1378. doi:10.3758/BF03206327

Berkowitsch, N. A. J., Scheibehenne, B., & Rieskamp, J. (in press). Rigorously testing Multialternative Decision Field Theory against standard random utility models. *Journal of Experimental Psychology: General*.

Berkowitsch, N. A. J., Scheibehenne, B., Rieskamp, J., & M. Matthäus (2013). Generalized Distance Function for Preferential Choices. Manuscript submitted for publication.

Berkowitsch, N. A. J., Scheibehenne, B., & Rieskamp, J. (2013). *How Previous Choices Affect Present Choices: Explaining Choice Dependencies with a Sequential Sampling Model*. Unpublished manuscript, Department of Psychology, University of Basel, Switzerland.

Bowler, S., Karp, J. A., & Donovan, T. (2010). Strategic coalition voting: Evidence from New Zealand. *Electoral Studies*, *29*, 350–357. doi: 10.1016./j.electstud.2010.03.001

Browne, M. W. (2000). Cross-validation methods. *Journal of Mathematical Psychology, 44*, 108–132. doi: 10.1006/jmps.1999.1289

Busemeyer, J. R., Barkan, R., Mehta, S., & Chaturvedi, A. (2007). Context effects and models of preferential choice: Implications for consumer behavior. *Marketing Theory, 7*, 39–58. doi: 10.1177/1470593107073844

Busemeyer, J. R., & Diederich, A. (2010). *Cognitive modeling.* Los Angeles, CA: Sage.

Busemeyer, J. R., & Townsend, J. T. (1993). Decision field theory: A dynamic-cognitive approach to decision making in an uncertain environment. *Psychological Review, 100*, 432–459.

Busemeyer, J. R., & Wang, Y. (2000). Model comparisons and model selections based on generalization criterion methodology. *Journal of Mathematical Psychology, 44*, 171–189. doi: 10.1006jmps.1999.1282

Carmon, Z., & Simonson, I. (1998). Price-quality trade-offs in choice versus matching: New insights into the prominence effect. *Journal of Consumer Psychology, 7*, 323–343.

Clotfelter, C. T., & Cook, P. J. (1993). Notes: The "Gambler's Fallacy" in lottery play. *Management Science*, *39*, 1521–1525. doi:10.1287/mnsc.39.12.1521

Conway, P., & Peetz, J. (2012). When does feeling moral actually make you a better person? Conceptual abstraction moderates whether past moral deeds motivate consistency or compensatory behavior. *Personality and Social Psychology Bulletin*, *38*, 907–919. doi:10.1177/0146167212442394

Daganzo, C. (1980). *Multinomial probit*. New York, NY: Academic Press.

Effron, D. A., Miller, D. T., & Monin, B. (2012). Inventing racist roads not taken: The licensing effect of immoral counterfactual behaviors. *Journal of Personality and Social Psychology*, *103*, 916–932. doi:10.1037/a0030008

Gigerenzer, G. (1996). Rationality: Why social context matters. In P. Baltes & U. M. Staudinger (Eds.), *Interactive minds: Life-span perspectives on the social foundation of cognition* (pp. 319–346). Cambridge, England: Cambridge University Press

Gluck, M. A., & Bower, G. H. (1988). From conditioning to category learning: an adaptive network model. *Journal of Experimental Psychology General*, *117*, 227–247.

Gluth, S., Rieskamp, J., & Büchel, C. (2012). Deciding when to decide: Time-variant sequential sampling models explain the emergence of value-based decisions in the human brain. *Journal of Neuroscience, 32*, 10686–10698. doi:10.1523/JNEUROSCI.0727-12.2012

Gneezy, A., Imas, A., Brown, A., Nelson, L. D., & Norton, M. I. (2012). Paying to be nice: Consistency and costly prosocial behavior. *Management Science, 58*, 179–187.

Green, P. E., & Srinivasan, V. (1990). Conjoint analysis in marketing: New developments with implications for research and practice. *Journal of Marketing*, *54*, 3–19.

Guadagno, R. E., & Cialdini, R. B. (2010). Preference for consistency and social influence: A review of current research findings. *Social Influence*, *5*, 152–163. doi:10.1080/15534510903332378

Johnson, J. G. (2006). Cognitive modeling of decision making in sports. *Psychology of Sport and Exercise, 7*, 631–52. doi: 10.1016/j.psychsport.2006.03.009

Hausman, J. A., & Wiese, D. A. (1978). A conditional probit model for qualitative choice: Discrete decisions recognizing interdependence and heterogeneous preferences. *Econometrica, 46*, 403–426.

Hey, J. D. (2001). Does repetition Improve consistency? *Experimental Economics, 4*, 5–54.

Highhouse, S. (1996). Context-dependent selection: The effects of decoy and phantom job candidates. *Organizational Behavior and Human Decision Processes*, *65*, 68–76. doi: 10.1006/obhd.1996.0006

Huber, J., Payne, J. W., & Puto, C. (1982). Adding asymmetrically dominated alternatives: Violations of regularity and the similarity hypothesis. *Journal of Consumer Research, 9*, 90–98.

Huber, J., & Puto, C. (1983). Market boundaries and product choice: Illustrating attraction and substitution effects. *Journal of Consumer Research, 10*, 31–44.

Jordan, J., Mullen, E., & Murnighan, J. K (2011). Striving for the moral self. The effects of recalling past moral actions on future moral behavior. *Personality and Social Psychology Bulletin, 37*, 701–713. doi: 10.1177/0146167211400208

Khan, U., & Dhar, R. (2006). Licensing effect in consumer choice. *Journal of Marketing Research*, *43*, 259–266. doi:10.2307/30163392

Kruschke, J. K. (1992). ALCOVE: An exemplar-based connectionist model of category learning. *Psychological Review*, *99*, 22–44.

Kruschke, J. K. (2011a). Bayesian assessment of null values via parameter estimation and model comparison. *Perspectives on Psychological Science, 6*, 299–312. doi: 10.1177/1745691611406925

Kruschke, J. K. (2011b). *Doing Bayesian data analysis*. *A tutorial with R and BUGS*. Burlington, MA: Academic Press.

Kruschke, J. K., Aguinis, H. & Joo, H. (2012). The time has come. Bayesian methods for data analysis in the organizational sciences. *Organizational Research Methods, 15*, 722–752. doi: 10.1177/1094428112457829

Lewandowsky, S., & Farrell, S. (2011). *Computational modeling in cognition*. Thousand Oaks, CA: Sage.

Loureiro, M. L., & Umberger W. J. (2007). A choice experiment model for beef: What US consumer responses tell us about relative preferences for food safety, country-of-origin labeling and traceability. *Food Policy*, *32*, 496–514. doi: 10.1016/j.foodpol.2006.11.006

Luce, R. D. (1959). *Individual choice behavior*. Oxford, England: Wiley.

McClelland, J. L., & Rumelhart, D. E. (1981). An interactive acitivation model of context effects in letter perception: Part 1. An account of basic findings. *Psychological Review, 88*, 375–407.

McFadden, D. (1973). Conditional logit analysis of qualitative choice behavior. In P. Zarembka (Ed.), *Frontiers in econometrics* (pp. 105-142). New York, NY: Academic Press.

McFadden, D. (2001). Economic choices. *American Economic Review, 91*, 351–378.

Monin, B., & Miller, D. T. (2001). Moral credentials and the expression of prejudice. *Journal of Personality and Social Psychology*, *81*, 33–43. doi:10.1037/0022-3514.81.1.33

Mosteller, F., & Nogee, P. (1951). An experimental measurement of utility. *Journal of Political Economy, 59*, 371–404.

Nosofsky, R. M. (1984). Choice similarity and the context theory of classification. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 10,* 104–114.

Otter, T., Johnson, J., Rieskamp, J., Allenby, G. M., Brazell, J. D., Diederich, A., … Townsend, J. (2008). Sequential sampling models of choice: Some recent advances. *Marketing Letters, 19*, 255–267. doi: 10.1007/s11002-008-9039-0

Ratcliff, R. (1978). A theory of memory retrieval. *Psychological Review, 85*, 59–108. doi: 10.1037/0033-295X.85.2.59

Reutskaja, E., Nagel, R., Camerer, C. F., & Rangel, A. (2011). Search dynamics in consumer choice under time pressure: An eye-tracking study. *American Economic Review, 101*, 900–926.

Rieskamp, J., Busemeyer, J. R., & Mellers, B. A. (2006). Extending the bounds of rationality: Evidence and theories of preferential choice. *Journal of Economic Literature, 44*, 631–661.

Rieskamp, J., & Otto, P. E. (2006). SSL: A theory of how people learn to select strategies. *Journal of Experimental Psychology: General*, *135*, 207–236. doi:10.1037/0096-3445.135.2.207

Rooderkerk, R. P., Van Heerde, H. J., & Bijmolt, T. H. A. (2011). Incorporating context effects into a choice model. *Journal of Marketing Research, 48,* 767–780.

Roe, R. M., Busemeyer, J. R., & Townsend, J. T. (2001). Multialternative decision field theory: A dynamic connectionist model of decision making. *Psychological Review, 108,* 370–392. doi: 10.1037//0033-295X.108.2.370

Sachdeva, S., Iliev, R., & Medin, D. L. (2009). Sinning saints and saintly sinners: The paradox of moral self-regulation. *Psychological Science*, *20*, 523–528. doi:10.1111/j.1467-9280.2009.02326.x

Scheibehenne, B., Wilke, A., & Todd, P. M. (2011). Expectations of clumpy resources influence predictions of sequential events. *Evolution and Human Behavior, 32*, 326–333. doi:10.1016/j.evolhumbehav.2010.11.003

Schwarz, W. (2011). Compensating tendencies in penalty kick decisions of referees in professional football: Evidence from the German Bundesliga 1963-2006. *Journal of Sports Science, 29,* 441–447. doi: 10.1080/02640414.2010.538711

Sheng, S., Parker, A. M., & Nakamoto, K. (2005). Understanding the mechanism and determinants of compromise effects. *Psychology & Marketing, 22*, 591–609. doi: 10.1002/mar.20075

Shepard, R. N. (1987). Toward a universal law of generalization for psychological science. *Science, 237,* 1317–1323.

Simao, J., & Todd, P. M. (2002). Modeling mate choice in monogamous mating systems with courtship. *Adaptive Behavior*, *10*, 113–136. doi:10.1177/1059712302010002003

Simonson, I., & Tversky, A. (1992). Choice in context: Tradeoff contrast and extremeness aversion. *Journal of Marketing Research, 24*, 281–295.

Slovic, P., & Tversky, A. (1974). Who accepts Savage's axiom? *Behavioral Science, 19*, 368–373.

Stewart, N., & Brown, G. D. A. (2004). Sequence effects in the categorization of tones varying in frequency. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *30*, 416–430. doi:10.1037/0278-7393.30.2.416

Stewart, N., Brown, G. D. A., & Chater, N. (2002). Sequence effects in categorization of simple perceptual stimuli. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *28*, 3–11. doi:10.1037/0278-7393.28.1.3

Stone, M. (1974). Cross-validatory choice and assessment of statistical predictions. *Journal of the Royal Statistical Society, 36B*, 111–147.

Thaler, R. H., & Sunstein C.R. (2008). *Nudge: Improving decisions about health, wealth, and happiness*. New Haven, CT: Yale Univ. Press.

Thurstone, L. L. (1927). A law of comparative judgment. *Psychological Review, 101*, 266–270.

Todd, P. M. (2007). How much information do we need? *European Journal of Operational Research*, *177*, 1317–1332. doi:10.1016/j.ejor.2005.04.005

Townsend, J. T., & Ashby, F. G. (1983). *Stochastic modeling of elementary psychological processes.* Cambridge, England: Cambridge University Press.

Train, K. E. (2003). *Discrete choice methods with simulation*. Cambridge, England: Cambridge University Press.

Trueblood, J. S. (2012). Multialternative context effects obtained using an inference task. *Psychonomic Bulletin & Review, 19*, 9962–9968. doi: 10.3758/s13423-012-0288-9

Trueblood, J. S., Brown, S. D., & Heathcote, A. (2013). The multi-attribute linear ballistic accumulator model of decision-making. In M. Knauff, M. Pauen, N. Sebanz, & I. Wachsmuth (Eds.), *Proceeding of the 35th Annual Conference of the Cognitive Science Society.* Austin, TX: Cognitive Science Society.

Trueblood, J. S., Brown, S. D., Heathcote, A., & Busemeyer, J. R. (2013). Not just for consumers: Context effects are fundamental to decision making. *Psychological Science, 24*, 901–908. doi: 10.1177/0956797612464241

Tsetsos, K., Chater, N., & Usher, M. (2012). Salience driven value integration explains decision biases and preference reversals. *Proceedings of the National Academy of Sciences, 109,* 9659–9664. doi: 10.1073/pnas.1119569109

Tversky, A. (1972a). Choice by elimination. *Journal of Mathematical Psychology, 9*, 341–367.

Tversky, A. (1972b). Elimination by aspects: A theory of choice. *Psychological Review, 79*, 281–299.

Tversky, A., Sattah, S., & Slovic, P. (1988). Contingent weighting in judgment and choice. *Psychological Review, 95,* 371–384.

Tversky, A., & Simonson, I. (1993). Context-dependent preferences. *Management Science, 39*, 1179–1189.

Unsworth, N., Heitz, R. P., Schrock, J. C., & Engle, R. W. (2005). An automated version of the operation span task. *Behavior Research Methods, 37,* 498–505. doi:10.3758/BF03192720

Usher, M., & McClelland, J. L. (2004). Loss aversion and inhibition in dynamical models of multialternative choice. *Psychological Review, 111,* 757–769.

Vickers, D. (1970). Evidence for an accumulator of psychophysical discrimination. *Ergonomics, 13*, 37–58.

von Neumann, J., & Morgenstern, O. (1947). *Theory of games and economic behavior* (2nd ed.). Princeton, NJ: Princeton University Press.

Wedell, D. H. (1991). Distinguishing among models of contextually induced preference reversal. *Journal of Experimental Psychology, 17,* 767–778.

White, C. N., Ratcliff, R., Vasey, M., W., & McKoon, G. (2010). Using diffusion models to understand clinical disorders. *Journal of Mathematical Psychology, 54*, 39–52. doi: 10.1016/j.jmp.2010.01.004

Zhang, S., Cornwell, J. F. M., & Higgins, E. T. (2013). Repeating the past: Prevention focus motivates repetition, even for unethical decision. *Psychological Science*. Advance online publication. doi: 10.1177/0956797613502363

A Generalized Distance Function for Preferential Choices

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Abstract

Many cognitive theories of decision making predict that people do not evaluate options independently of each other. To what extent the preference for one option is influenced by other available options can depend on how similar the options are to each other, where similarity is assumed to be a decreasing function of the distance between options. We examine how the distance between preferential options with multiple attributes can be determined. Previous distance functions either ignore the importance weight people assign to the attributes, are limited to two attributes, or neglect the preferential relationship between the options. To measure the distance between preferential options it is necessary to take the subjective preferences of the decision maker into account. Accordingly, the multi-attribute space can be stretched or shrunk relative to the attention or importance that a person gives to different attributes. We propose a *generalized distance function* for preferential choices in the multi-attribute space that accounts for their preferential relationship and allows for individual differences according to subjective preferences. Using a hands-on example, we illustrate its application and compare the determined distances to previous distance measures. We conclude with a discussion of the suitability and limitations of the proposed distance function.

Keywords: distance; similarity; preferential choice; multi-attribute; decision making

1. Introduction

Past work has illustrated repeatedly that when people make choices, they do not evaluate options independently but rather evaluate options relative to each other (e.g., Huber, Payne, & Puto, 1982; Rieskamp, Busemeyer, & Mellers, 2006; Simonson & Tversky, 1992; Slovic & Tversky, 1974). One way to explain these interdependent evaluations is to assume distance-dependent competition of options in the multi-attribute space (Roe, Busemeyer, & Townsend, 2001; Rooderkerk, Van Heerde, & Bijmolt, 2011). Accordingly, options in the multi-attribute space compete with each other based on their perceived similarity, where similarity is defined as a decreasing function of their distance (Nosofsky, 1984; Shepard 1987). The more similar two options are, the more strongly the evaluation of one option will affect the evaluation of the other option. In the present article we propose a *generalized distance function* that defines the relationships between preferential choice options in a multi-attribute space and takes a person's subjective preferences into account.

The psychological distance between options cannot be defined simply by the Euclidean distance in the multi-attribute space, because this ignores the preferential relationship between options and the importance the decision maker gives to different attributes. Figure 1A illustrates this difference for the hypothetical choice between notebook computers: The labeled dots A, B, and C represent three different notebooks described by their processor speed and their battery life. Although the Euclidean distances between Notebooks A and C and Notebooks A and B in the multi-attribute coordinate space are exactly the same, we argue that their psychological distances differ. Notebooks A and B are highly competitive, as they both lie on the indifference line, meaning that a change from Notebook A to Notebook B appears acceptable because the loss in battery life is compensated by Notebook B’s higher processor speed. In comparison, Notebook C appears completely inferior to Notebooks A and B and a change from Notebooks A or B to Notebook C is not acceptable for the decision maker, because this change would lead to a loss of battery life and a loss of processor speed. In other words, Notebook C is *dominated* by Notebooks A and B. Consequently, Options A and B are perceived as more similar to each other than either is to option C, so that the psychological distance is smaller between A and B than between A and C (or B and C). One way to represent the psychological distance between the options is to define a distance along the dominance line and the indifference line, as suggested by Hotaling, Busemeyer, and Li (2010) and Huber et al. (1982). The two distances can be expressed by an *indifference vector* (dashed lines in Figure 1A and B) and a *dominance vector* (continuous lines in Figure 1A and B). In addition, the psychological distance between options also depends on the subjective importance given to the different attributes. A person giving equal importance to the two attributes might be indifferent about Notebooks A and B. However, a person giving higher importance to the attribute "battery life" as compared to the attribute "processor speed" will be indifferent only about a choice between Notebook A and Notebook B′ and not between Option A and Option B (see Figure 1A).

INSERT FIGURE 1 HERE.

This example illustrates that to be broadly applicable, a distance function needs to meet several requirements: It should capture the different preferential relationships between options (i.e., whether one option is dominant or not); it must be flexible enough to account for varying importance of different attributes; and it should be applicable for choice problems with more than two attributes. Past research on distance functions has addressed some of these requirements (Hotaling et al., 2010; Huber et al., 1982; Nosofosky, 1986; Rooderkerk et al., 2011; Wedell, 1991) but an approach incorporating all requirements simultaneously is missing. Our goal in the present article is to provide a generalized distance function for preferential choice options and to compare it to alternative approaches. An empirical test of the function in terms of a tradeoff between the functions' predictive accuracy compared to its complexity (e.g., Forster, 2000) is left to researchers incorporating a distance function into choice models. Without incorporating the function within a choice model it is difficult to access its predictive accuracy. In Section 2, we review approaches that offer partial solutions to the listed requirements. Based on these approaches, we develop a generalized distance function fulfilling all requirements simultaneously. We then illustrate the application of the generalized distance function with a real-life example. We conclude with a discussion of the suitability and limitations of the proposed distance function.

2. **Psychological distance**

Researchers studying perceptual categorization have emphasized the difference between the objective distance between objects and their psychological distance. Nosofsky (1986), for instance, argued that individuals do not distribute their attention equally to each dimension describing (perceptual) objects. That is, their psychological space differs. He suggested a weighted Minkowski metric, which stretches and shrinks the different dimensions relative to the attention an individual devotes to each dimension (Carroll & Wish, 1974). More attention to an attribute stretches the dimension in the space, whereas less attention shrinks the dimension in the space. This approach is standard in categorization research (Nosofsky & Johansen, 2000; Nosofsky & Zaki, 2002; Zaki, Nosofsky, Stanton, & Cohen, 2003). When applying this approach to preferential decision making it is necessary to take the preferential relationship between options into account; that is, one needs to distinguish distances in the indifference direction from distances in the dominance direction.

Huber et al. (1982) already advocated treating distance in the dominance and indifference directions differently. They suggested that "dominated items…are represented in the limit as being an infinite distance below those items that dominate them" (p. 92), whereas "distances among non-dominating pairs…must be finite" (p. 92). In other words, they proposed placing a higher weight on the distance in the dominance direction relative to the distance in the indifference direction. Figures 1A and B illustrate this idea. The distance between Options A and B is the same in both graphs, whereas the distance between Options A and C is larger in Figure 1B than in Figure 1A, because the psychological space is stretched in the dominance direction.

In his dimensional weight model Wedell (1991) suggested rotating the indifference and dominance vectors to account for the preferential relationship when a dominating option is added to the choice set. The very same logic could be applied to account for varying levels of attribute importance. First, consider an individual who weights the two attributes processor speed and battery life for the notebook example equally. This can be expressed by choosing an indifference dimension and a dominance dimension along the diagonals of the Cartesian coordinate plane (see the solid indifference and dominance vectors *iv* and *dv* in Figure 2). Accordingly, an individual who weights processor speed higher (lower) than battery life would be depicted by a steeper (less steep) indifference vector *iv′* (*iv″*); see Figure 2.

INSERT FIGURE 2 HERE.

In their preferential choice model, Rooderkerk and colleagues (2011) discussed an alternative approach to account for individual differences. They suggested the indifference vector is the same for all individuals, whereas the dominance vector (a.k.a. preference vector) is multiplied by an individual weight, yielding dominance vectors of different lengths. We think that the rotation approach (i.e., rotating indifference vector) from Wedell (1991) that we follow here has some advantages over Rooderkerk et al.'s (2011) suggestion of individual dominance vectors: The indifference vector directly expresses how many units an individual is willing to give up to increase the other attribute by one unit without introducing an additional parameter to weight the dominance vector.

This overview of past research reveals that several approaches to distance functions have addressed some of the aforementioned requirements, representing necessary steps toward a generalized distance function for preferential choices, but none of them fulfill all requirements simultaneously (see Table 1). In the following we provide a generalized distance function that does just that. We first describe it conceptually before providing the mathematical details.

INSERT TABLE 1 HERE.

3. A generalized distance function

We continue with the notebook example with two attributes (i.e., processor speed, battery life) before generalizing it to multiple attributes. As a first step we need to assure that the attributes are comparable to each other, to avoid distortions due to different scales and ranges of attribute values. This is achieved by normalizing the attribute values to the same range, for example between 0 and 10, which additionally increases the visual interpretability of the indifference vector. Next, we have to specify the directions and lengths of the indifference and dominance vectors. Because the dominance vector is orthogonal to the indifference vector (Tversky, Sattah, & Slovic, 1988; Wedell, 1991), the direction of the dominance vector results from the indifference vector. The direction of the indifference vector can be determined by so-called “exchange ratios,” which indicate how many units of one attribute an individual is willing to give up to increase the other attribute by one unit. That is, the indifference vector contains information about the exchange ratios between the attributes. For options described by more than two attributes the number of possible exchange ratios increases. These multiple exchange ratios can be captured by *multiple indifference vectors* forming an indifference plane.

For example, all possible exchange ratios of an option described by five attributes can be captured with four exchange ratios, resulting in four indifference vectors—one fewer than the number of attributes (for details see Section 4). Whereas the number of indifference vectors depends on the number of attributes, the number of distance types remains constant: Options are still described by *distance in the indifference direction* and *distance in the dominance direction*. An option either dominates another option or it does not, which means that we still have a single dominance vector even for several attributes. As a consequence, a single parameter is sufficient to weight distance in the dominance direction more strongly than in the indifference direction.

As is the case for two attributes (i.e., one indifference vector), the dominance vector has to be orthogonal to all indifference vectors in the multi-attribute case. Because of this property the dominance vector can be derived from the indifference vectors (see Section 4 for details). To obtain indifference vectors and a dominance vector of equal lengths, we simply normalize them to the Euclidean length of 1, by dividing each vector by its Euclidean length.

As a next step we express the distance between two options by means of the distance in the indifference and dominance direction. The line connecting the two options is called the distance vector, represented in the standard attribute coordinate plane. Instead of using these attribute dimensions, we now express the distance vector in terms of the indifference vectors and the dominance vector. This is achieved by a change of basis. The transformed distance vector indicates how many units of the standardized indifference vectors and of the standardized dominance vector are necessary to "travel" from one to another option.

Finally, we calculate the Euclidean length of the transformed distance vector and multiply distance in the dominance direction by a parameter higher than 1. If this parameter is set to 1 then the resulting distance equals the Euclidean distance of the unweighted distance vector.

4. Mathematical formalization of the generalized distance function

To specify the generalized distance function we first rescale the attribute values (v_{old}) of the *n* attributes to equal ranges, from min_{new} to max_{new} according to

$$v_{\text{new}} = \text{min}_{\text{new}} + \frac{(v_{\text{old}} - \text{min}_{\text{old}}) \cdot (\text{max}_{\text{new}} - \text{min}_{\text{new}})}{\text{max}_{\text{old}} - \text{min}_{\text{old}}}. \tag{1}$$

Next, we define an importance weight vector $\boldsymbol{W}$ that contains the individual importance weights for the *n* attributes and scale the weights such that they sum to 1. The number of possible exchange ratios can then be calculated according to

$$n_{\text{ExchRat}} = \frac{n(n-1)}{2}, \tag{2}$$

where n is the number of attributes. A strategy to reduce the needed n_{ExchRat} without the loss of information is to compare each attribute against an arbitrary attribute, for example, against the first attribute. Therefore, each indifference vector $\{\boldsymbol{iv_j}\}_{j=1}^{n-1}$ is an n-dimensional vector and can be calculated as

$$\boldsymbol{iv_j} = \begin{bmatrix} -\frac{w_{j+1}}{w_1} \\ 0 \\ \vdots \\ 0 \\ \frac{w_1}{w_1} \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{w_{j+1}}{w_1} \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \forall\, j = 1, \dots, n-1, \tag{3}$$

where 1 is at the $(j + 1)^{\text{th}}$ position.

Notice that this leads to $n - 1$ indifference vectors. Independently of the number of attributes, each $\boldsymbol{iv_j}$ has exactly two nonzero entries, the exchange ratios. This is because the other n_{ExchRat} can be recovered from the entries of the $n - 1$ indifference vectors. The first entry in each $\boldsymbol{iv_j}$ marks the exchange ratio between the $(j + 1)^{\text{th}}$ and the first attribute and the algebraic sign indicates that the vector is pointing toward the axes of the $(j + 1)^{\text{th}}$ attribute. In other words, $\boldsymbol{iv_j}$ reveals how many units of the $(j + 1)^{\text{th}}$ attribute are gained for giving up on one unit of the first attribute.

Because the n-dimensional dominance vector $\boldsymbol{dv}$ should be orthogonal to all $n - 1$ indifference vectors, we determine the case in which the dot product of each indifference vector to $\boldsymbol{dv}$ is zero. In general, this vector fulfills

$$\boldsymbol{iv_j} \cdot \boldsymbol{dv} = 0, \forall\, j = 1, \dots, n-1, \tag{4}$$

which leads to the generalized form

$$\boldsymbol{dv} = \begin{bmatrix} \frac{w_1}{w_1} \\ \frac{w_2}{w_1} \\ \vdots \\ \frac{w_j}{w_1} \\ \vdots \\ \frac{w_n}{w_1} \end{bmatrix}. \tag{5}$$

Now we can build the $n \times n$ matrix $\boldsymbol{B}^*$, containing the $n - 1$ indifference vectors $\boldsymbol{iv_1}$ to $\boldsymbol{iv_{n-1}}$ and the dominance vector $\boldsymbol{dv}$. It is

$$\boldsymbol{B}^* = [\boldsymbol{iv_1}, \dots, \boldsymbol{iv_j}, \dots, \boldsymbol{iv_{n-1}}, \boldsymbol{dv}]. \tag{6}$$

Observe that $\boldsymbol{B}^*$ is a basis of the attribute space. To standardize the lengths of the indifference vectors and the dominance vector to 1, each vector is divided by its Euclidean lengths l_{iv} and l_{dv}, where $\left\{l_{\mathrm{iv}_j}\right\}_{j=1}^{n-1}$

$$l_{iv_j} = \left\|\boldsymbol{iv_j}\right\|_2, \forall\, j = 1, \dots, n-1 \tag{7}$$

and

$$l_{dv} = \|\boldsymbol{dv}\|_2. \tag{8}$$

Thus, we obtain the basis $\boldsymbol{B}$, which is

$$\boldsymbol{B} = \left[\frac{\boldsymbol{iv_1}}{l_{iv_1}}, \dots, \frac{iv_j}{l_{iv_j}}, \dots, \frac{\boldsymbol{iv_{n-1}}}{l_{iv_{n-1}}}, \frac{\boldsymbol{dv}}{l_{dv}}\right]. \tag{9}$$

$\boldsymbol{B}$ contains the standardized indifference vectors and the standardized dominance vector. Next, we define the standard distance vector $\boldsymbol{dist_{stand}}$ as the trajectory path between two points, expressed in standard unit vectors. We want to transform this distance vector into the new distance vector $\boldsymbol{dist_{trans}}$ that expresses the trajectory path created by the previously introduced indifference vectors and by the dominance vector. This is achieved by a change of basis. Therefore we multiply the inverse of the basis $\boldsymbol{B}$ by $\boldsymbol{dist_{stand}}$:

$$\boldsymbol{dist_{trans}} = \boldsymbol{B^{-1}} \cdot \boldsymbol{dist_{stand}}. \tag{10}$$

The first $n - 1$ entries of $\boldsymbol{dist_{trans}}$ express the distance in units of each $\boldsymbol{iv_j}$, whereas the last entry of $\boldsymbol{dist_{trans}}$ expresses the distance in units of $\boldsymbol{dv}$. Now we need to calculate the Euclidean length $\boldsymbol{D^2}$ of $\boldsymbol{dist_{trans}}$ and multiply the distance in the dominance direction by a parameter $\boldsymbol{wd} > 1$. This assures that the distance in the dominance direction is weighted more strongly than the distance in the indifference directions. This is computed as follows:

$$\boldsymbol{D^2} = \boldsymbol{dist_{trans}}' \cdot \boldsymbol{A} \cdot \boldsymbol{dist_{trans}}, \tag{11}$$

where $\boldsymbol{A}$ is an $n \times n$ diagonal matrix and is constructed in the following way:

$$\boldsymbol{A_{j,j}} = \left\{ \begin{matrix} 1, & if\ j = 1, \ldots n-1 \\ \boldsymbol{wd}, & if\ j = n \end{matrix} \right\}. \tag{12}$$

This assures that only the difference in dominance direction—the last column of $\boldsymbol{dist_{trans}}$—is weighted by $\boldsymbol{wd}$. By setting $\boldsymbol{A}$ to the identity matrix (i.e., $\boldsymbol{wd} = 1$), one obtains the standard Euclidean norm.

Taken together, the steps required to derive the generalized distance function can be summarized as follows:

1. Normalize all attribute values to the same range (for example between 0 and 10).
2. Determine the weights $\boldsymbol{W}$ to calculate the matrix $\boldsymbol{B}^*$, containing the $n - 1$ indifference vectors and the dominance vector.
3. Normalize the indifference vectors and the dominance vector in $\boldsymbol{B}^*$ to the Euclidean length of 1 to obtain $\boldsymbol{B}$.
4. Express the distance vector $\boldsymbol{dist_{stand}}$ in terms of the introduced basis $\boldsymbol{B}$ to get the transformed distance vector $\boldsymbol{dist_{trans}}$.
5. To obtain $\boldsymbol{D^2}$, determine the Euclidean length of $\boldsymbol{dist_{trans}}$ and place a higher weight on distance in the dominance direction relative to the indifference direction.

In Section 5, we provide an example in which we apply the generalized distance function to calculate the distances between three notebooks, each described by three attributes, and compare the obtained distances to previous distance functions.

5. Example

Let us consider the following example. Imagine you want to buy a new notebook computer to conduct your research. The notebook should be fast enough to run your simulations smoothly and the battery life should cover your daily commuting time by train. During your internet search you came across notebooks with processors up to 5 GHz and batteries that last for 6 h, and you excluded notebooks with display sizes larger than 30 in. Table 2 shows the three notebooks you ended up with.

INSERT TABLE 2 HERE.

Let us further assume that you care about having a high processor speed (PS) as much as a long battery life (BL), but you care less about having a large display size (DS). This might be represented by the following importance weights: $w_{\mathrm{PS}} = 0.4$, $w_{\mathrm{BL}} = 0.4$, and $w_{\mathrm{DS}} = 0.2$. Therefore, you clearly prefer Notebooks A and B over Notebook C since they dominate Notebook C. The choice between Notebooks A and B seems to be harder. Going back and forth between the two notebooks you realize that both have some advantages and disadvantages. You deem the two notebooks equally attractive. That is, you are indifferent about Notebooks A and B. Because Notebook A dominates Notebook C and competes with Notebook B, the perceived distance between Notebooks A and C should be larger than between Notebooks A and B. However, if we determined the standard Euclidean distance in the rescaled attribute space, the two distances would be the same, as illustrated below.

To apply the generalized distance function to the notebooks, we start by normalizing the ranges of all three attributes ($n = 3$) to equal ranges, for instance, between 0 and 10 according to Equation 1. Now, the notebooks can be represented as points in the multi-attribute space, where Notebook A = (5, 4, 3), Notebook B = (3, 5, 5), and Notebook C = (3, 2, 2). Next, we determine the basis $\boldsymbol{B}^*$ containing the two indifference vectors and the dominance vector. The first indifference vector $\boldsymbol{iv_1}$ is given by

$$\boldsymbol{iv_1} = \begin{bmatrix} -\frac{w_{\mathrm{BL}}}{w_{\mathrm{PS}}} \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix},$$

the second indifference vector $\boldsymbol{iv_2}$ is given by

$$\boldsymbol{iv_2} = \begin{bmatrix} -\frac{w_{\mathrm{DS}}}{w_{\mathrm{PS}}} \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \\ 1 \end{bmatrix},$$

and the dominance vector $\boldsymbol{dv}$ is given by

$$\boldsymbol{dv} = \begin{bmatrix} 1 \\ \frac{w_{\mathrm{BL}}}{w_{\mathrm{PS}}} \\ \frac{w_{\mathrm{DS}}}{w_{\mathrm{PS}}} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0.5 \end{bmatrix}.$$

This leads to the basis $\boldsymbol{B}^*$:

$$\boldsymbol{B}^* = \begin{bmatrix} -1 & -0.5 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0.5 \end{bmatrix}.$$

Next, we normalize the indifference vectors and the dominance vector to the length of 1 by dividing them by their Euclidean lengths, where

$$l_{\boldsymbol{iv}_1} = \sqrt{(-1)^2 + 1^2 + 0} = \sqrt{2},$$

$$l_{\boldsymbol{iv}_2} = \sqrt{(-0.5)^2 + 0 + 1^2} = \sqrt{1.25},$$

and

$$l_{\boldsymbol{dv}} = \sqrt{1^2 + 1^2 + 0.5^2} = \sqrt{2.25}.$$

Then the standardized basis $\boldsymbol{B}$ is

$$\boldsymbol{B} = \begin{bmatrix} \frac{-1}{\sqrt{2}} & \frac{-0.5}{\sqrt{1.25}} & \frac{1}{\sqrt{2.25}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2.25}} \\ 0 & \frac{1}{\sqrt{1.25}} & \frac{0.5}{\sqrt{2.25}} \end{bmatrix}.$$

To express the distance vector $\boldsymbol{dist_{stand}}$ between the notebooks in terms of the indifference vector and the dominance vector, we next make a change of basis. The standard distance vectors between Notebooks A and C, and Notebooks A and B are

$$\boldsymbol{dist_{standAC}} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

and

$$\boldsymbol{dist_{standAB}} = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}.$$

To yield the transformed distance vector we apply Equation 10, so that

$$\boldsymbol{dist_{transAC}} = \begin{bmatrix} \frac{-1}{\sqrt{2}} & \frac{-0.5}{\sqrt{1.25}} & \frac{1}{\sqrt{2.5}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2.5}} \\ 0 & \frac{1}{\sqrt{1.25}} & \frac{0.5}{\sqrt{2.5}} \end{bmatrix}^{-1} \cdot \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

and

$$\boldsymbol{dist_{transAB}} = \begin{bmatrix} \frac{-1}{\sqrt{2}} & \frac{-0.5}{\sqrt{1.25}} & \frac{1}{\sqrt{2.5}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2.5}} \\ 0 & \frac{1}{\sqrt{1.25}} & \frac{0.5}{\sqrt{2.5}} \end{bmatrix}^{-1} \cdot \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} -\sqrt{2} \\ -\sqrt{5} \\ 0 \end{bmatrix}.$$

The distance $\boldsymbol{dist_{transAC}}$ indicates that to reach Point A from Point C, we need to move three units along the dominance vector and none along the two indifference vectors. To reach Point A from Point B, we need to move by $-\sqrt{2}$ and $-\sqrt{5}$ units along the first and second indifference vectors, and no units in the dominance direction, which means that we are moving on the indifference plane.

By setting $\boldsymbol{wd}$ = 10, we assume that distance in the dominance direction is perceived 10 times more strongly than in the indifference direction, resulting in

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix}.$$

Finally, we can calculate the Euclidean distances $\boldsymbol{D^2}_{\boldsymbol{AC}}$ between Options A and C and $\boldsymbol{D^2}_{\boldsymbol{AB}}$ between Options A and B by applying Equation 11. We obtain $\boldsymbol{D^2}_{\boldsymbol{AC}} = 90$ and $\boldsymbol{D^2}_{\boldsymbol{AB}} = 7$. If we had treated distances in the indifference and dominance directions equally (i.e., by setting $\boldsymbol{A}$ to identity), the distance between Options A and C would have decreased to

$\boldsymbol{D^2}_{\boldsymbol{AC}} = 9$, while $\boldsymbol{D^2}_{\boldsymbol{AB}}$ would have remained 7, giving the impression that Options B and C have similar distances to Option A. Note that if we also had ignored the weights given to the attributes and had determined the Euclidean distance in the rescaled attribute space, both Notebook C and Notebook B would have had the same distance to Notebook A of $\boldsymbol{D^2}_{\boldsymbol{AC\ Eucl.}} = 9\ (= 2^2 + 2^2 + 1^2)$ and $\boldsymbol{D^2}_{\boldsymbol{AB\ Eucl.}} = 9\ (= 2^2 + (-1)^2 + (-2)^2)$, incorrectly suggesting that Notebooks A and C are perceived as being as similar to each other as Notebooks A and B are to each other.

6. Discussion

People often evaluate options relative to each other (e.g., Huber et al. 1982; Rieskamp, et al., 2006; Simonson & Tversky, 1992; Slovic & Tversky, 1974). Therefore, many cognitive models of decision making take the similarity between options into account (e.g., Roe et al., 2001; Rooderkerk, et al., 2011). The similarity between options is generally expressed as a decreasing function of their distance to each other (Shepard, 1987). Past research has followed different approaches to define the distance between options for preferential decision-making problems. These approaches have tackled different important aspects of preferential choice, such as the different preferential relationships between options, the varying importance of different attributes, and the applicability to choice problems with more than two attributes. However, none of the suggested approaches has addressed all of these requirements simultaneously (cf. Hotaling et al. 2010; Huber et al., 1982; Nosofosky, 1986; Rooderkerk et al., 2011; Wedell, 1991). Therefore, we developed a new generalized distance function for preferential choices that distinguishes the preferential relationship between options in the multi-attribute space, assigns different weights depending on the type of distance, and accounts for the individual degree of importance of different attributes. We compared the proposed function to other alternative approaches, but did not test them empirically against each other in terms of a tradeoff of its predictive accuracy compared to its complexity (e.g., Forster, 2000), as this was not the goal of the present paper.

There are several assumptions that we made to define the generalized distance function. First, we followed a specific approach to express the importance of different attributes. In particular, we assumed that people can express the importance of one attribute relative to another attribute by linear exchange ratios between the attributes. This implies that a disadvantage on one unit for one attribute can always be compensated for by a specific number of units of an advantage on the other attribute. However, this assumption might sometimes be violated. For instance, a decision maker might require a minimum battery life for her new notebook. In this case she would do better to apply nonlinear exchange ratios. Here, for instance, the closer a value of an attribute gets to a decision maker's required minimum value (e.g., minimum hours of battery life), the more units of the other attributes are required to compensate a further decrease of the first attribute. In the extreme, the exchange ratio becomes indefinitely small or big, which can be captured by asymptotic indifference curves. This idea is supported by findings from Chernev (2004). He suggested that due to people's extremeness aversion, they prefer options that are closer to the so-called "attribute-balance line," which he defined as the line connecting "all potential options with identical values on both attributes" (p. 251). This can be captured by a convex indifference curve. However, for simplicity we have applied linear exchange ratios to express the importance of different attributes. It remains to be tested whether nonlinear exchange ratios lead to a substantial advantage in describing the psychological distance for multi-attribute options in preferential decision making and ultimately to better prediction of preferential choices.

Second, when we normalized the indifference vectors and the dominance vector to equal lengths of 1, and later when we calculated the length of the transformed distance vectors, we assumed the 2-norm distance (i.e., the Euclidean distance). Although 2-norm distance is widely applied for preferential choices (e.g., Hotaling et al., 2010; Rooderkerk et al. 2011), the 1-norm (i.e., the city block) distance has been frequently applied in categorization research for stimuli with highly separable dimensions (e.g., Nosofsky & Zaki, 2002). For example, one hypothesis following from this is that with an increasing number of attributes describing a preferential choice option, the dimensions become less separable and the 2-norm can outperform the 1-norm. This can also be tested—similarly to the Minkowski *r* metric—by introducing the norm as a free parameter *r* to see whether the 1-norm (i.e., $r = 1$) or the 2-norm (i.e., $r = 2$) leads to more accurate predictions of preferential choices.

The approach we have taken to define a generalized distance function shares some aspects with the multidimensional scaling approach (MDS; Kruskal, 1964a, 1964b). MDS is often applied in marketing to optimally visualize the perceived similarity between options in the attribute space (for a review see Carroll & Arabie, 1980). MDS also allows the definition of a new attribute space to express the similarities between options. However, our approach takes the specific preferential relationship between options into account, is applied to the perception and decision of a single individual.

Besides fulfilling its general purpose of measuring the perceived distance between preferential choice options, the generalized distance function is particularly useful for studying so-called context effects (Huber et al., 1982; Huber & Puto, 1983; Simonson & Tversky, 1992; Slovic & Tversky, 1974). Context effects refer to choice situations in which preferential choice options are evaluated relative to each other. According to the similarity effect—one context effect—similar choice options compete with each other more strongly than dissimilar options. Therefore adding an option that is similar to one but dissimilar to another option increases the preference for the dissimilar relative to the similar option (Tversky, 1972a, 1972b). To better understand the similarity effect it is therefore important to determine how similar or dissimilar two options are perceived to be. Whereas two options might be perceived as similar for one person, for another person one of the two options might dominate the other. This can be captured by the generalized distance function we proposed, because it accounts for individual differences and distinguishes the preferential relationship between options. Research on context effects has focused mainly on options described by two attributes. One reason is that with an increasing number of attributes, it gets harder to tell dominated options apart from indifferent options. Because the generalized distance function allows for multi-attribute options, it can be applied to examine context effects in the multi-attribute space. Furthermore, the generalized distance function can be applied to study individual differences, since it allows the indifference vectors to rotate through the multi-attribute space. From their directions one can infer the individuals exchange ratios between the attributes. Because of this property the generalized distance function can be incorporated into choice models that try to estimate the subjective importance people give to different attributes.

In sum, people often evaluate options relative to each other. Therefore, to explain how people make preferential choices it is important to define how similar options are to each other. For defining similarity we proposed a generalized distance function. This function simultaneously accounts for several requirements previous approaches have addressed in isolation. This allows researchers to study preferential choices and context effects in more depth and to investigate individual differences in the multi-attribute space. Ultimately, this should lead to advancement of decision theory by taking the similarity between choice options into account for providing better explanations and predictions of human preferential choices.

References

Carroll, J. D., & Arabie, P. (1980). Multidimensional scaling. *Annual Review of Psychology, 31*, 607-649.

Carroll, J. D., & Wish, M. (1974). Models and methods for three-way multidimensional scaling. In D. H. Krantz, R. C. Atkinson, R. D. Luce, & P. Suppes (Eds.), *Contemporary developments in mathematical psychology* (Vol. 2, pp. 57-105). San Francisco, CA: Freeman.

Chernev, A. (2004). Goal orientation and consumer preference for the status quo. *Journal of Consumer Research, 31*, 557-565.

Forster, M. R. (2000). Key concepts in model selection: Performance and generalizability. *Journal of Mathematical Psychology, 44*, 205-231.

Hotaling, J. M., Busemeyer, J. R., & Li, J. (2010). Theoretical developments in decision field theory: Comment on Tsetsos, Usher, and Chater. *Psychological Review, 117*, 1294-1298. doi: 10.1037/a0020401

Huber, J., Payne, J. W., & Puto, C. (1982). Adding asymmetrically dominated alternatives: Violations of regularity and the similarity hypothesis. *Journal of Consumer Research, 9*, 90-98.

Huber, J., & Puto, C. (1983). Market boundaries and product choice: Illustrating attraction and substitution effects. *Journal of Consumer Research, 10*, 31-44.

Kruskal, J. B. (1964a). Multidimensional scaling by optimizing goodness of fit to a nonmetric hypothesis. *Psychometrika, 29*, 1-27.

Kruskal, J. B. (1964b). Nonmetric multidimensional scaling: A numerical method. *Psychometrika, 29*, 115-129.

Nosofsky, R. M. (1984). Choice similarity and the context theory of classification. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 10*, 104-114.

Nosofsky, R. M. (1986). Attention, similarity, and the identification-categorization relationship. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 14*, 700-708.

Nosofsky, R. M., & Johansen, M. K. (2000). Exemplar-based accounts of "multiple-system" phenomena in perceptual categorization. *Psychonomic Bulletin & Review, 7*, 375-402.

Nosofsky, R. M., & Zaki, S. R. (2002). Exemplar and prototype models revisited: Response strategies, selective attention, and stimulus generalization. *Journal of Experimental Psychology: Learning, Memory and Cognition, 28*, 924-940. doi: 10.1037/0278-7393.28.5.924

Rieskamp, J., Busemeyer, J. R., & Mellers, B. A. (2006). Extending the bounds of rationality: Evidence and theories of preferential choice. *Journal of Economic Literature, 44*, 631-661.

Roe, R. M., Busemeyer, J. R., & Townsend, J. T. (2001). Multialternative decision field theory: A dynamic connectionist model of decision making. *Psychological Review, 108*, 370-392. doi: 10.1037//0033-295X.108.2.370

Rooderkerk, R. P., Van Heerde, H. J., & Bijmolt, T. H. A. (2011). Incorporating context effects into a choice model. *Journal of Marketing Research, 48*, 767-780.

Shepard, R. N. (1987). Toward a universal law of generalization for psychological science. *Science, 237*, 1317-1323.

Simonson, I., & Tversky, A. (1992). Choice in context: Tradeoff contrast and extremeness aversion. *Journal of Marketing Research, 29*, 281-295.

Slovic, P., & Tversky, A. (1974). Who accepts Savage's axiom? *Behavioral Science, 19*, 368-373. doi: 10.1002/bs.3830190603

Tversky, A. (1972a). Choice by elimination. *Journal of Mathematical Psychology, 9*, 341-367.

Tversky, A. (1972b). Elimination by aspects: A theory of choice. *Psychological Review, 79*, 281-299.

Tversky, A., Sattah, S., & Slovic, P. (1988). Contingent weighting in judgment and choice. *Psychological Review, 95*, 371-384.

Wedell, D. H. (1991). Distinguishing among models of contextually induced preference reversal. *Journal of Experimental Psychology, 17*, 767-778.

Zaki, S. R., Nosofsky, R. M., Stanton, R. D., & Cohen, A. L. (2003). Prototype and exemplar accounts of category learning and attentional allocation: A reassessment. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 29*, 1160-1173. doi: 10.1037/0278-7393.29.6.1160

Table 1

Comparison of Distance Functions

Approach	Preferential relationship between options	Higher weight on distances in dominance relative to indifference direction	Importance weighting of attributes	Multiple attributes
Huber et al. (1982)	✓	✓	–	–
Nosofosky (1986)	–	–	✓	✓
Wedell (1991)	–	–	✓	–
Hotaling et al. (2010)	✓	✓	–	–
Rooderkerk et al. (2011)	✓	–	✓	–
Generalized distance function	✓	✓	✓	✓

Table 2

Attribute Values of Three Different Notebook Computers

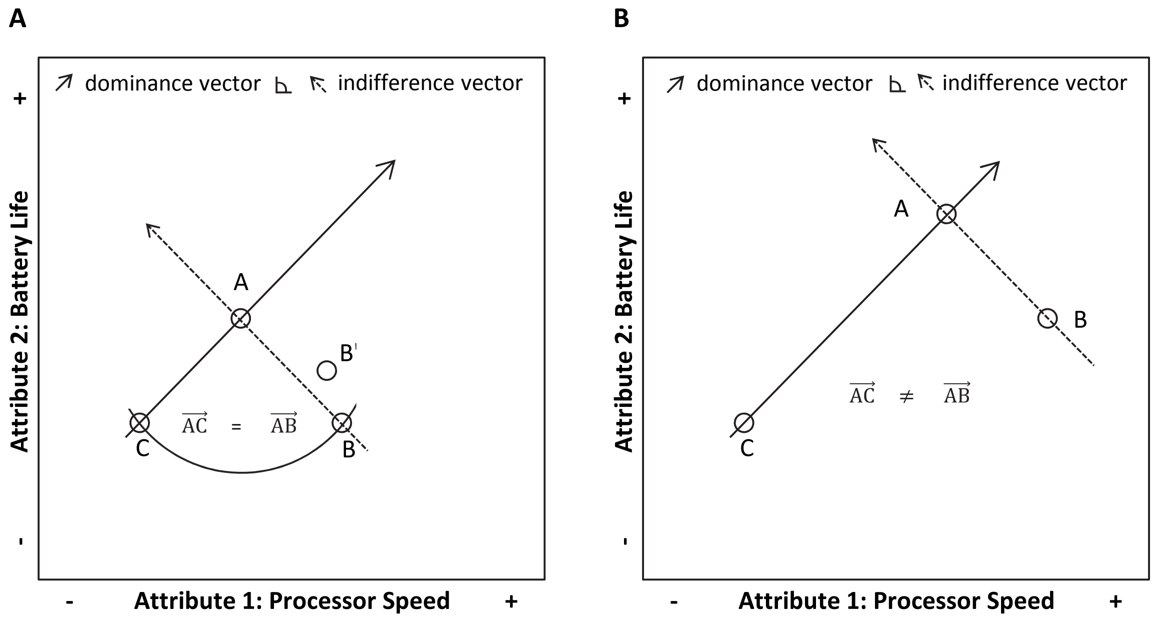
Attribute	Notebook A	Notebook B	Notebook C
Processor speed in gigahertz [1–5]	3.0	2.2	2.2
Battery life in hours [2–6]	3.6	4.0	2.8
Display size in inches [4–30]	11.8	17.0	9.2

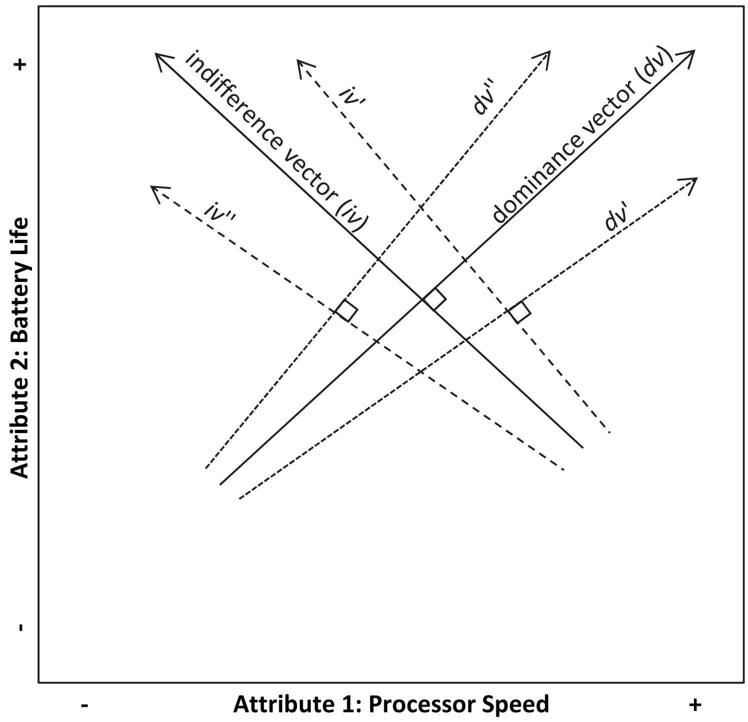
[a] The numbers in brackets indicate the possible value range.

Figure Captions.

Figure 1. The Euclidean distance (A) and the psychological distance (B) for the same set of notebook computers, labeled A to C. (A) The two options B and C have the same Euclidean distance but not the same psychological distance as Option A. The psychological distance of Option C to Option A and Option B is relatively large, because Option C is dominated by the other two options. A person giving higher importance to the attribute “battery life” as compared to the attribute “processor speed” will be indifferent only about a choice between Notebook A and Notebook B′ and not between Option A and Option B, resulting in a rotated indifference vector (see also Figure 2). (B) The psychological space is stretched in dominance direction, whereas distance in the indifference direction is the same as in (A).

Figure 2. Individual preferences in a two-attribute space illustrated by an individual indifference vector and an orthogonal dominance vector. The solid indifference and dominance vectors indicate a person who weights the two attributes “processor speed” and “battery life” equally, whereas the dashed indifference and dominance vectors indicate a person giving a higher weight to either processor speed (*iv′* and *dv′*) or battery life (*iv″* and *dv″*).


A
dominance vector
indifference vector
A
B'
C
B
$\overrightarrow{AC} = \overrightarrow{AB}$
Attribute 2: Battery Life
Attribute 1: Processor Speed
+
-
-
+
B
dominance vector
indifference vector
A
B
C
$\overrightarrow{AC} \neq \overrightarrow{AB}$
Attribute 2: Battery Life
Attribute 1: Processor Speed
+
-
-
+


indifference vector (iv)
iv'
dv''
dominance vector (dv)
iv''
dv'
Attribute 2: Battery Life
+
-
Attribute 1: Processor Speed
-
+

Rigorously Testing Multialternative Decision Field Theory Against Random Utility Models

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Abstract

Cognitive models of decision making aim to explain the process underlying observed choices. Here, we test a sequential sampling model of decision making, Multialternative Decision Field Theory (MDFT; Roe, Busemeyer, & Townsend, 2001) on empirical grounds and compare it against two established random utility models of choice, the probit and the logit model. Using a within-subject experimental design, participants in two studies repeatedly choose among sets of options (consumer products) described on several attributes. The results of Study 1 show that all models predicted participants' choices equally well. In Study 2, where the choice sets were explicitly designed to distinguish the models, MDFT had an advantage in predicting the observed choices. Study 2 further revealed the occurrence of multiple context effects within single participants, indicating an interdependent evaluation of choice options and correlations between different context effects. In sum, the results indicate that sequential sampling models can provide relevant insights into the cognitive process underlying preferential choices and thus can lead to better choice predictions.

Keywords: preferences, process models, MDFT, random utility models, context effects, Bayesian statistics

Testing Multialternative Decision Field Theory Rigorously Against Random Utility Models

Recently, cognitive models of decision making aiming for a better explanation of human behavior by describing the processes underlying observed choices have received increasing attention. In contrast to this, many existing models of decision making do not account for cognitive processes but rather focus on just predicting observable outcomes. Surprisingly, so far, most comparisons between these outcome-oriented, static models against cognitive process models have been made on theoretical grounds and rigorous tests of these models against each other have rarely been conducted on empirical grounds. One reason for this lack of empirical comparisons in the decision-making literature may be the difficulty of estimating the free parameters of many cognitive process models. Whether these models provide a feasible alternative thus remains somewhat unclear. Here, we propose a testable version of Multialternative Decision Field Theory (MDFT, Roe, Busemeyer, & Townsend, 2001), a prominent cognitive process model of choice, and compare it on empirical grounds to two established and widely used random utility models (RUMs) of choice that make no cognitive process assumptions but merely aim to predict decision outcomes. The main goal of the present work is to provide a rigorous empirical test of MDFT against the standard RUMs and illustrate people's interdependent evaluations of preferential choice options.

When trying to predict the outcome of a decision, one common approach relies on the theoretical framework of expected utility (von Neumann & Morgenstern, 1947). Provided that people's preferential choices obey certain choice axioms, this framework allows constructing a utility function such that their choices represent expected utility maximization. Due to this axiomatic approach, expected utility theories make deterministic predictions and cannot account for the probabilistic character of human choice (e.g., Mosteller & Nogee, 1951; Rieskamp, 2008). To account for randomness in people's choices, expected utility theory has been extended with an explicit error theory, leading to RUMs. Although RUMs do not aim for a description of the underlying cognitive processes that lead to the observable decision outcomes, they allow predicting the probability with which options are chosen (e.g., McFadden, 2001; Train, 2003). Standard RUMs assume that options are evaluated independently, such that the utility of any single option does not depend on other available options in the choice set.

Perhaps the two most prominent and widely used RUMs are the (multinomial) logit and probit models (e.g., Daganzo, 1980; Hausman & Wiese, 1978; Luce, 1959; McFadden, 1973; Thurstone, 1927). Both models have a long success record and are frequently applied in economics, psychology, consumer research, and related fields, including the domains of travel behavior (e.g., Adamowicz, Louviere, & Williams, 1994; Hensher, 1994; Train, 1978; Wardman, 1988), environmental behavior (e.g., Hanley, Wright, & Adamowicz, 1998; Roberts, Boyer, & Lusk, 2008), political choice behavior (e.g., Bowler, Karp, & Donovan, 2010; Karp, 2009; Nownes, 1992), consumer choices (e.g., Green & Srinivasan, 1978, 1990), or food choices (e.g., Gil & Sánchez, 1997; Loureiro & Umberger, 2005, 2007). For example, using probit models, Ryan and Farrar (2000) analyzed preferences in health care (e.g., treatment in a local clinic vs. treatment in a hospital) and Phillips, Maddala, and Johnson (2002) measured preferences for different HIV tests. Further, Loureiro and Umberger (2007) analyzed the importance U.S. consumers assign to the country-of-origin labeling and traceability of beef, and Koistinen et al. (2013) investigated the impact of fat content and carbon footprint information on the relative preferences of Finns for minced meat—both using logit models. Presumably, the widespread use of these models is largely due to their ease of implementation and estimation (Train, 2003).

In contrast to these outcome-oriented models, many cognitive approaches to decision making aim for a description of the processes that underlie observable choices (e.g., Busemeyer & Diederich, 2002; Lewandowsky & Farrell, 2011). Within this category, sequential sampling models represent a particularly promising approach (e.g., Busemeyer &

Townsend, 1993; Scheibehenne, Rieskamp, & Gonzalez-Vallejo, 2009; Usher & McClelland, 2004). These models have a long tradition in psychology, explaining for instance, memory and perception processes (e.g., Ratcliff, 1978; Townsend & Ashby, 1983; Vickers, 1970). Sequential sampling models of preferential choice often assume that people accumulate information or evidence about the available options over time and that a choice is made once the accumulated evidence passes a decision threshold. One sequential sampling model that has been suggested as a powerful theory for modeling preferential choices is MDFT (Roe et al., 2001). MDFT aims to explain how preferences are formed and how they evolve over time. The model assumes that at any point in time during the deliberation process a preference is formed for each available option until the accumulated evidence of one option reaches a predefined decision threshold. Each temporary preference state represents the integration of all previous states. The preferences are formed based on an attention-switching process that assumes attention between the attributes of the options (e.g., the price or the quality of consumer products) switches from one point in time to the next in an all-or-nothing manner. The integration of all previous preference states is subject to a decay function that accounts for imperfect preference recollection such that parts of the previous preference states are lost during the integration (Johnson & Busemeyer, 2010). MDFT further assumes that at each point in time options are evaluated relative to each other by comparing the attribute values. Finally, the theory assumes that options inhibit each other as an increasing function of their similarity, related to the concept of lateral inhibition (McClelland & Rumelhart, 1981). That is, closer (i.e., more similar) options inhibit each other more strongly than more distant options.

Besides having the ability to advance the theoretical understanding of cognitive processes, MDFT also promises a higher predictive accuracy, because it takes situational aspects into account such as time pressure, cognitive load, or similarities between options that RUMs ignore (e.g., Diederich, 2010; Pettibone, 2012; Roe et al., 2001). Such a cognitive and process-driven approach has often been called for in the choice literature (e.g., Chandukala, Kim, Otter, Rossi, & Allenby, 2007; Otter et al. 2008; Reutskaja, Nagel, Camerer, & Rangel, 2011). Although promising, complex cognitive models with several free parameters such as MDFT are inherently more flexible in fitting any observed data compared to more parsimonious RUMs such as the logit and probit models. Therefore, the question arises whether sequential sampling models such as MDFT still yield an increase in predictive accuracy when model complexity is taken into account.

Past research indicated that MDFT can explain a number of systematic violations of standard RUMs based on theoretical grounds (e.g., Busemeyer, Barkan, Mehta, & Chaturvedi, 2007; Rieskamp, Busemeyer, & Mellers, 2006). However, rigorous comparisons of these models on empirical grounds are lacking. Thus, it is an open empirical question to what extent the increased model complexity of MDFT actually yields improved predictive accuracy as compared to the more parsimonious RUMs. An empirical test will also clarify whether the frequent application of RUMs in many domains is justified. Such a test requires that MDFT's free parameters can be estimated from empirical data. Estimating a model using empirical data differs qualitatively from illustrating specific predictions from a set of given parameter values. Surprisingly, to our knowledge, despite MDFT's prominence, empirical studies designed to estimate the model's parameters in empirical studies are lacking. Presumably, this is because in its original form, the choice probabilities that MDFT predicts are not analytically specified but need to be derived from time-consuming process simulations. Similarly, Otter et al. (2008, p. 259) pointed out that due to "specification issues and computational challenges" the estimation process of MDFT on the basis of empirical data is difficult (see also Soltani, De Martino, & Camerer, 2012).

In the following, we meet this challenge by providing a version of MDFT that can be applied to observed choice data. Based on this, we describe an empirical study in which we tested and compared MDFT against two of the presumably most prevalent RUMs in the choice literature, the logit and the probit model. We tested the models in choice situations in which participants repeatedly chose consumer products out of a set of three options (Study 1). In Study 2 we tested and compared the models in choice situations that entailed so-called context effects (described in detail below), which presumably allow MDFT to perform in its highest gear. We provide a detailed description of MDFT and how to estimate its parameters based on empirical data. In the next section we provide a specification of the logit and probit models.

The Logit and Probit Models

The logit and probit models both assume that options are compared based on their respective subjective utilities and that the option with the highest utility is most likely chosen. The two models differ in their error theories, which lead to differences in the options' utilities. For a single decision maker, the utility of an option *i* out of a set of *J* options is defined as

$$U_i = V_i + \varepsilon_i \qquad (1)$$

where V_i indicates the subjective value of that option and the error term ε_i represents a random variable with $\varepsilon = [\varepsilon_1, \ldots \varepsilon_i, \ldots \varepsilon_J]$. The logit model assumes that the error ε is extreme value distributed whereas the probit model assumes normally distributed errors (Train, 2003). For both models, the subjective value V_i is the product of a vector β that contains the weights (i.e., the importance) given to the *m* attributes of the option, and the value vector X_i with $X = [X_1, \ldots X_i, \ldots X_J]$ containing the attribute values of option *i*

$$V_i = \beta X_i \,. \qquad (2)$$

The probability *p* of choosing option *i* is defined as

$$p_i = \text{probability}\,(V_i + \varepsilon_i > V_j + \varepsilon_j ; \forall\, j \neq i)$$

$$= \int_\varepsilon I(V_i + \varepsilon_i > V_j + \varepsilon_j ; \forall\, j \neq i) f(\varepsilon)\, d\varepsilon \qquad (3)$$

where $I(\cdot)$ is an indicator function (see Train, 2003 for details).

For the logit model, but not the probit model, a closed form representation of this integral exists:

$$p_i = \frac{e^{V_i}}{\sum_{j \neq i}^{J} e^{V_j}}. \tag{4}$$

Thus, the logit model has m free parameters, one for each attribute weight. To make the probit model mathematically identifiable, different parameterizations exist (Train, 2003). Here, we chose to fix one of the weight parameters, which leaves the probit model with $m - 1$ attribute weight parameters and one free parameter v that specifies the variance of the normal distributed error term ε.

Multialternative Decision Field Theory

In MDFT, the preference for each option at any point in time t is captured by a preference vector P_t referred to as a preference state containing the preferences of all J options. P_t integrates all previous preference states and adds the current evaluation or valence V_t of the options according to the following updating process:

$$P_t = SP_{t-1} + V_t. \tag{5}$$

The process described by Equation 5 continues until one option reaches a predefined decision threshold (a so-called internal stopping rule) or when the decision time is up (external stopping rule). Here, S is a feedback matrix that reflects to what extent the previous preference states for the given options are memorized (diagonal elements) and how the options influence each other depending on their distances in the attribute space (off-diagonal elements). It is defined as

$$S = \delta - \varphi_2 \times \exp(-\varphi_1 \times D^2) \tag{6}$$

where δ is an identity matrix, the decay parameter φ_2 determines the diagonal elements of S, and the sensitivity parameter φ_1 determines the similarity as a function of the distance D between the options in the attribute space (cf. Hotaling, Busemeyer, & Li, 2010). MDFT assumes that people evaluate each option relative to each other option; that is, people compare an option's attribute value with the corresponding value of the other option on that attribute. This process leads to interdependent evaluations of choice options. This process is reflected in the valence vector V_t that can be decomposed into three matrices and an error component

$$V_t = CMW_t + \varepsilon \tag{7}$$

where C is a contrast matrix to compute the advantages or disadvantages of each option relative to the alternative options (see Busemeyer & Diederich, 2002 for the general formula of C). The value matrix M contains the attribute values of each option (comparable to the value matrix X in the logit and probit models), and the weight vector W_t represents the attention or importance weights for each attribute. Over time, each attribute informs the valence formation proportional to its importance (comparable to the weight vector β in the logit and probit models). If the updating process is omitted, the probit model becomes a special case of MDFT; that is, it is nested within MDFT (see Supplemental Materials for the mathematical details). For the full description of how to determine MDFT's choice probabilities see also Appendix B of Roe et al. (2001).

Estimating MDFT

To estimate the free parameters of MDFT based on observed choice data one must solve Equation 5. Unless certain auxiliary assumptions are imposed, this requires laborious numerical integration techniques as analytic solutions are not readily available (see also Trueblood, Brown, & Heathcode, 2013; Tsetsos, Usher, & Chater, 2010). Furthermore, as the predictions of MDFT depend on the similarities between the options in the attribute space, a distance function must be specified (Hotaling et al., 2010; Tsetsos et al., 2010). To address these requirements, we made the simplifying assumption that decision makers only decide once the preference state for the different options stabilizes; that is, converges to a specific value. Therefore we set $t \to \infty$.[1] Although this approach sacrifices MDFT's ability to make predictions about decision time, no decision threshold or maximum decision time needs to be specified and an analytical solution to calculate the choice probabilities exists (see Appendix A for the mathematical derivations).

As mentioned above, MDFT assumes that preferences partly depend on the similarity of the available options. To determine the effect of these similarities, a function is needed that quantifies these influences. Existing distance functions (e.g., Hotaling et al., 2010) assume equal weighting of the attributes or consider a maximum of only two attributes. To allow for more than two attributes and flexible attribute weights, we included a generalized distance function that describes the distance between any two options in the multidimensional space with indifference vector(s) and a dominance vector (Berkowitsch, Scheibehenne, Matthäus, & Rieskamp, 2013). The indifference vectors specify how much importance a person gives to the different attributes, and the dominance vector specifies the preferential relationship between the options; that is, whether an option dominates another option or not (Hotaling et al., 2010; Tversky, Sattah, & Slovic, 1988; Wedell, 1991). The underlying idea here is that the psychological distance decreases more slowly when moving from one option to another along the line of preferential indifference (i.e., the indifference direction), than along the line of preferential dominance (i.e., the dominance direction). To account for different weighting of the dominance and indifference directions, only the dominance vector is multiplied by a weight $wd > 1$, as suggested by Huber, Payne, and Puto (1982) and also by Hotaling et al. (2010; for more details on the generalized distance function, see Appendix B and Berkowitsch et al., 2013).

Taken together, this specification of MDFT requires the estimation of $m - 1$ attention weight parameters for each product attribute, the variance parameter v of the normally distributed error component, the rate parameter φ_1 at which similarity declines with distance between the options, and the decay parameter φ_2 specifying how quickly the preference state decays during the updating process. In Study 1 the dominance parameter wd was not estimated because in this study dominated options were eliminated from the choice sets. In contrast, in Study 2 in which dominated options were part of the choice sets, the parameter was estimated (see Appendix C for the constraints on the parameters). Implemented this way, for example, a choice between three options with five attributes requires the estimation of seven (eight with *wd*) parameters, as compared to five free parameters for both the logit and the probit model.

Study 1: Comparing MDFT to RUMs

Study 1 aims to test whether the model parameters of MDFT can be estimated from observed choice data when certain simplifying assumptions are met (see above). This requirement is necessary for comparing the predictive accuracy of MDFT to that of alternative choice models such as RUMs on empirical grounds, which was another goal of Study 1. Toward these goals, we conducted two consecutive experiments in which participants repeatedly chose their favorite digital camera out of a set of three available options. In the first calibration experiment, we compared how well the models could describe participants' choice behavior. In the second generalization experiment, we used the results from the calibration experiment to create new choice sets for which MDFT and the RUMs made maximally different predictions. This generalization test allows for a rigorous comparison between the two models that takes model complexity into account without the need to re-estimate the models' parameters (Busemeyer & Wang, 2000).

Method

Participants. Thirty university students (66% female, mean age 24 years) participated in each of the two experiments. The average participant took about 13 and 15 min, respectively, to complete the experiments and received a show-up fee of 3 Swiss francs (3 USD; calibration experiment) and 10 Swiss francs (11 USD; generalization experiment).

Procedure and design. To incentivize participants' choices, all participants were entered into a lottery where they had a chance to win one of the cameras they chose in the experiment (or a very similar one). Each camera was described by five attributes: megapixels

(4, 6, or 8), optical zoom (3×, 5×, or 10×), picture quality (good vs. very good), screen size (2 vs. 3 inch) and availability of optical image stabilizer (yes vs. no). This led to a total of 72 distinct cameras that were presented in 72 different randomized sets of three cameras each. To create the choice sets for both the calibration and the generalization experiment, we first created all possible 357,840 (i.e., 72×71×70) sets of three cameras. Next we deleted all sets with dominant options. From the remaining pool we randomly selected 72 choice triplets.

Based on participants' choices in the calibration experiment, we estimated the models' parameters for each individual participant. Based on these parameters, we selected 72 new choice triplets for which the models made maximally different predictions. To select this generalization set, we generated predictions from each model for 1,000 randomly sampled choice triplets. Model predictions were generated using bootstrap methods in which we randomly sampled (with replacement) sets of parameters from the participants in the calibration experiment (see Busemeyer & Wang, 2000). Triplets for the generalization experiment were selected by finding those 72 triplets for which the average city block distance between the models' predictions was highest both between MDFT and the logit model and between MDFT and the probit model. The city block distance sums the absolute difference of the predicted mean probability between the models for each option, thus providing a single distance value for each choice triplet (Attneave, 1950).

Model comparison. As a first comparison step in the calibration experiment, we estimated the free parameters of all models using maximum likelihood methods. The search space for the parameter values was restricted within a reasonable range (see Appendix C for details of the range for each parameter). To take the models' complexities into account we determined the Bayesian information criterion (BIC) for each model (Raftery, 1995). We used the difference in BIC values to determine the relative posterior probability that a model generated the data for each individual participant. These probabilities were scaled so that they added up to 1 across models (Raftery, 1995). Because MDFT has more free parameters, it gets penalized more strongly than either the logit or the probit model. Additionally, we tested the models against a baseline model assuming a random choice between the three options with a probability of 1/3. Naturally, any reasonable model is expected to out-compete the baseline model in predicting the observed choices. In the next step, we applied a generalization test to compare the accuracies of the models' predictions for the generalization experiment that were based on the estimated models' parameters from the calibration experiment (Busemeyer & Wang, 2000).

Results

Descriptive results. To investigate the agreement between participants' choices within each of the two experiments, we calculated the relative frequency of the most popular option within each of the 72 choice triplets. On average, 66% and 58% of the participants chose the same digital camera in each triplet in the calibration and generalization experiment, respectively.

Model comparison calibration experiment. The mean log-likelihood μ_{LL} (with μ_{LL} = 0 indicating perfect fit) across all participants was highest for MDFT (μ_{LL} = -48.65), followed by the logit model (μ_{LL} = -50.00) and the probit model (μ_{LL} = -50.57), whereas the μ_{LL} of the baseline model was -79.10, illustrating that all three models made more accurate predictions than the baseline model. Comparisons of the log-likelihoods on an individual level indicated that 22 (73%) participants were best described by MDFT. The remaining eight (27%) participants were best described by the logit model and nobody was best described by the probit model.

Figure 1 shows the results of the model comparison based on the BIC. Here, relative model probabilities were classified as weak (.33–.60), positive (.60–.91), strong (.91–.99), and very strong (> .99; adapted from Raftery, 1995, for three models). The figure illustrates that when taking the models' complexities into account, the advantage of MDFT over the RUMs diminishes: Now, 52% (25) of participants were best described by the logit model, followed by 8% best described by the probit model and 2% by MDFT.

As an additional illustrative measure of absolute model fit, we examined the percentage of choices in which the option with the highest predicted probability was chosen by the participants. The percentages of correctly predicted choices were comparable across models (MDFT: 74%, logit: 73%, probit: 72%) and considerably higher than the baseline model (33%).

The correlations between the estimated attribute weights across models indicate that all three models yield comparable attribute weights, $r_{\text{logit,probit}} = .82$ ($SD = .38$), $r_{\text{MDFT,probit}} = .79$ ($SD = .38$), $r_{\text{MDFT,logit}} = .94$ ($SD = .17$).[2] This result indicates that all three models were able to identify the importance the participants gave to the different attributes (see the Supplemental Materials for the estimated ranges of the parameters). For instance, on average, the attribute of picture quality (w_3) was identified as having the largest importance for digital cameras by all three models (Tables S1-S3).

Model comparison generalization experiment. As an initial measure of out-of-sample accuracy, we analyzed how well each of the models predicted choices in the generalization experiment. For all but five choices the three models agreed in their prediction of what option would most likely be chosen. Therefore, the mere percentage of how often the most frequently chosen option was predicted correctly was quite similar (MDFT: 82%, probit: 81%, logit: 75%). Nevertheless and decisively, the probabilities by which the models predicted the choices differed substantially. To take these differences into account we compared the models based on the mean log-likelihood of their probabilistic predictions. Here, MDFT ($\mu_{LL} = -6.81$) and the probit model ($\mu_{LL} = -6.50$) were most accurate, whereas the baseline ($\mu_{LL} = -9.79$) and the logit model were least accurate ($\mu_{LL} = -11.99$). Analyses on the basis of single choices indicated that MDFT and the probit model predicted people's choices most accurately in 31 (43%) and 32 (44%) of the 72 choice triplets, respectively. A closer look at the predictions for each triplet revealed that, on average, the choice probabilities predicted by the logit model were most extreme—resulting in either very accurate or very inaccurate predictions. For the five triplets where the three models made qualitatively different predictions, MDFT predicted participants' choices most accurately, as indicated by the mean LL for these five options (μ_{LL_MDFT} = -10.61, μ_{LL_probit} = -14.29, μ_{LL_logit} = -16.44, and $\mu_{LL\_baseline}$ = -16.09).

Discussion of Study 1

In Study 1 we outlined how MDFT could be estimated on the basis of empirical data and we tested it against two competing models in two consecutive choice experiments. In the first calibration experiment, MDFT provided a better fit to the observed choice data for most participants. However, when taking model complexity into account, the advantage of MDFT over the more parsimonious RUMs largely disappeared, suggesting that the advantage of MDFT was mainly due to its higher flexibility in fitting the data. The crucial second generalization experiment revealed that MDFT outperformed the logit model and rivaled the probit model, surpassing the latter for cases in which MDFT made qualitatively different predictions. Thus, when predicting preferential choices, MDFT was not necessarily better but also not worse than the RUMs in the context at hand.

As outlined in the introduction, one advantage of MDFT over RUMs is that it can take systematic influences of the context and similarities between the available options into account. Previous research has identified a number of situations in which option evaluations systematically depended on the context of other available options (Huber et al., 1982; Huber & Puto, 1983; Simonson & Tversky, 1992; Slovic & Tversky, 1974). These findings violate the assumption that options are evaluated independently, sometimes referred to as the independence of irrelevant alternatives (IIA) principle (cf. Rieskamp et al., 2006). According to this principle, the ratio of the choice shares of any two options stays constant when another option is added or removed from the set of options (Luce, 1959). Systematic violations of the

IIA principle can be elicited by selectively adding options to an existing set of options (e.g., Huber et al., 1982; Huber & Puto, 1983; Tversky 1972a, 1972b). In contrast to RUMs, MDFT provides a cognitive explanation of when and how systematic violations of the IIA principle occur and in theory it can predict these violations. Thus, in a choice situation where systematic context effects are likely to occur, MDFT should outperform RUMs.

Because of the way we created the choice sets in Study 1, we did not expect any systematic violations of the IIA principle. Thus, even though the experiment may resemble common choice situations in real life and in the lab, presumably it did not allow MDFT to perform in its highest gear. Therefore, the following study tested MDFT against RUMs for situations in which violations of the IIA principle were expected to occur frequently.

Study 2: Comparing MDFT to RUMs Using Context Effects

Three well-known context effects that systematically violate the IIA principle are the so-called attraction, compromise, and similarity effects (described in more detail below). There is a substantial body of research on these effects (for a review see Heath & Chatterjee, 1995), yet so far, almost all empirical studies have focused on each of the single effect in isolation or used a between-subjects experimental design to elicit the effects; however, there are no theoretical reasons for this separation,. If these context effects are due to interdependent evaluations of choice options, multiple context effects should also occur for the same person. Indeed, using an inference and a perceptual task, Trueblood and colleagues reported empirical findings showing that all three context effects can occur within the same experimental design (Trueblood, 2012; Trueblood, Brown, Heathcote, & Busemeyer, 2013) and within a single person (Trueblood, Brown, & Heathcote, 2013). Similarly, Tsetsos, Chater, and Usher (2012) elicited the attraction and the similarity effect within individuals using a risky choice task. Therefore, it seems plausible that comparable effects could also occur for preferential choices, which is the domain in which context effects presumably have received most attention in the past. To find out, the present study aims to test whether all three context effects can occur simultaneously for the same person in a preferential choice task. In the following, we describe the three context effects in more detail:

The *attraction effect* refers to a choice situation in which adding an option dominated by one of the existing options increases the choice share of the dominating option (Huber et al., 1982; Huber & Puto, 1983). This effect additionally represents a violation of the so-called regularity principle, according to which the absolute choice share of an option can only stay constant or decrease when a new option is added to a set of options. Another well-documented context effect is the *compromise effect*, which can occur when a third option is added to an existing set such that one of the original options appears as a compromise, thereby increasing its relative choice share (Simonson & Tversky, 1992; Tversky & Simonson 1993). Finally, the *similarity effect* is based on the observation that adding a choice option that is similar to one but not to the other option has been shown to increase the relative choice share of the dissimilar option, presumably because the similar options “compete” more strongly with each other (Tversky, 1972a; 1972b).

Model Comparison

In theory, MDFT can simultaneously account for all three of these context effects by incorporating different cognitive mechanisms given a specific set of parameter values (Roe et al., 2001). However, it is unclear whether this specific set of parameters is also suitable to accurately predict people’s preferential choices. In the present study we sought to test how well MDFT can predict empirical choice data when people’s preferential choices are affected by different context effects. Like in Study 1, the main question in this situation was again whether MDFT provides a better explanation of the data as compared to RUMs that cannot account for systematic context effects. Thus, we predicted that the advantage of MDFT over RUMs would increase as decision makers became more prone to context effects.

To test the advantage of MDFT over RUMs in cases where multiple context effects occur within a single individual, we created choice sets that increased the chances of observing attraction, compromise, and similarity effects within the same person. As outlined in detail below, we did this by systematically varying the position of choice options in the attribute space. We compared the models against each other based on BIC and cross-validation (Browne, 2000; Stone, 1974).

Method

Participants. Forty-eight students (67% female, mean age 24 years) at the University of Basel participated in Study 2 in exchange for 25 Swiss francs (CHF). The study took 57 min on average.

Procedure and design. Participants repeatedly chose a consumer product described on two attributes. The study consisted of two consecutive sessions. The first session aimed to find pairs of options to which individuals were indifferent. In the second session, we systematically added new choice options to that initial pair to elicit different context effects within each participant.

To identify indifference pairs in the first session, participants repeatedly filled in missing attribute values (e.g., price) so that two products (e.g., a heavier and a lighter racing bike) became equally attractive (Carmon & Simonson, 1998). Prior to this matching task, we provided participants with a short explanation of the attributes and the possible value range for each of the six products (see Figure 2 for an example of this task and Table 1 for a list of the utilized products and their attributes). With this matching procedure we created 108 pairs of options (i.e., 18 per product pair) to which each single participant was expected to be indifferent. We refer to these pairs as the "matched" options.

The second session took place a few days later and involved a choice task that was similar to Study 1. For each participant, the previously matched Options A and B were combined with one new option that was carefully placed within the attribute space (see Figure 3).[3] We balanced the positions of the new options, such that participants were faced with six attraction, compromise, and similarity triplets for each matched pair. We refer to the matched option expected to be chosen more frequently on theoretical grounds as the *target*. The remaining matched option is referred to as the target's *competitor*. We balanced the choice triplets for each participant such that for half of the choice triplets the target was Option A (Figure 3a) and for the other half it was Option B (Figure 3b).

For attraction triplets (BDA and DBA in Figure 3a and b, respectively) the dominated option was placed at a distance of about 10% from the target, orthogonal to the indifference line $\overline{\mathrm{AB}}$. For the compromise triplets (BAC and CBA in Figure 3a and b, respectively) the extreme option was placed along the indifference line such that the target had the same distance to the competitor and to the extreme option. We chose a distance of about 10% of $\overline{\mathrm{AB}}$ in the indifferent direction between the competitor and the new similar option to create similarity triplets (SBA and BAS in Figure 3a and b, respectively). Due to rounding, half of the intended similarity choice triplets became attraction choice triplets (i.e., the added option did not lie on the indifference line) and were therefore excluded from the analysis, which reduced the statistical power to analyze the similarity effect. However, the design was still balanced in the sense that for half of the triplets Option A was the target, and for the other half Option B was the target.

Using this balanced design, we compared the relative choice shares of the three options and subsequently tested for each participant whether adding a new option to the previously matched options influenced the *relative choice share of the target (RST),* defined as

$$RST = \frac{N\,\text{targets}}{N\,\text{targets} + N\,\text{competitors}} \tag{8}$$

where "*N* targets" indicates how often a participant chose the target option and "*N* competitors" indicates how often the competitor was chosen. The measure was calculated separately for the attraction, compromise, and similarity triplets. Due to the way the choice sets were created, the target and the competitor will be chosen about equally often if no context effect occurs (i.e., *RST* = .50). An *RST* value larger than .50 indicates an increase of the target choice share relative to that of the competitor, and hence a systematic context effect. The order of the products as well as the order of the choice triplets within products was randomized.

Results

Occurrence of context effects. Did adding the third option to the previously matched pairs change people's preferences? Figure 4 shows a histogram of the choice shares of the target, the competitor, and the added option, averaged across all participants and products, separately for the attraction, the compromise, and the similarity triplets. As can be seen from the figure, on a descriptive level the *RST* of all three types of triplets exceeded .50.

Next, we tested whether the mean *RST* across all participants was larger than .50 for any of the three context situations. As a statistical measure, we calculated the 95% highest posterior density interval (HDI) representing the most credible *RST* values using Bayesian statistics (Kruschke, 2011a, 2011b). If the HDI_{95} excluded .50 one can infer a reliable context effect. We adapted a hierarchical Bayesian model following Kruschke (2011b) assuming uniform prior probability distributions across the parameter range.

Results of that analysis indicated a strong and reliable effect for the attraction triplets with an HDI_{95} of .60–.66 (mean: .63). Essentially, 100% of the posterior probability density was above the critical threshold of .50. Likewise, for the compromise triplets a reliable effect was observed with an HDI_{95} of .52–.63 (mean: .58) with 99% of the density being above .50. For the similarity triplets the HDI_{95} of .48–.59 (mean: .54) included the critical value of .50, although 93% of the posterior density was above .50, suggesting a weak similarity effect.

Are participants who are prone to one context effect also prone to another? In total, nine participants (19%) showed an *RST* value higher than .50 for all three context effects. On an individual level, the individual HDI_{95} for each of these participants did not exclude .50 for all three context effects, which is probably due to having too little statistical power on the individual level. For example, if these nine participants are analyzed post hoc as a group of subjects, HDI_{95} would exclude .50 for all three effects. Figure 5 plots the *RST* across individual participants. The figure shows that the *RST* was positively correlated for the attraction and compromise triplets (r = .49*, SD* = .10), indicating that participants who showed the attraction effect also often showed the compromise effect. Interestingly, there was a strong negative correlation for the *RST* between the similarity and the attraction triplets (r = -.53*, SD* = .10) and between the similarity and the compromise triplets (r = -.58*, SD* = .15). This indicates that people who showed either the attraction or the compromise effect rarely showed the similarity effect.

Model comparison. To compare the predictive accuracies of MDFT against RUMs, we estimated the free parameters of the models using maximum likelihood techniques similar to Study 1. Results indicated that the mean log-likelihood across participants was lowest for MDFT (μ_{LL} = -57.81), followed by the logit model (μ_{LL} = -66.49) and the probit model (μ_{LL} = -71.08). All three models predicted the observed choices better than a baseline model predicting random choices (μ_{LL} = -91.21). Comparing the log-likelihoods within each individual revealed that MDFT outperformed both the logit and the probit model for each of the 48 participants.

Additional analyses based on the mean log-likelihood across participants and triplets revealed the highest difference between MDFT and the RUMs occurred for attraction triplets (MDFT: μ_{LL} = -0.61, logit: μ_{LL} = -0.73, probit: μ_{LL} = -0.80), followed by compromise triplets (MDFT: μ_{LL} = -0.74, logit: μ_{LL} = -0.86, probit: μ_{LL} = -0.90), and a weaker difference for similarity triplets (MDFT: μ_{LL} = -0.81, logit: μ_{LL} = -0.84, probit: μ_{LL} = -0.89).

To test whether MDFT still outperforms the RUMs if the models' complexities are taken into account, we calculated the relative model probabilities based on the BIC, similar to Study 1. This analysis revealed that the choice behavior of 31 participants (65%) was best described by MDFT, and 12 participants (25%) were best described by the logit model. The remaining 5 participants (10%) were best described by the probit model. For a substantial number of participants, the obtained relative model probabilities indicated very strong evidence for MDFT. In contrast, for those participants who were best described by the logit model, the evidence was mostly weak. Figure 6 summarizes these results.

To test whether the 13 participants for whom the relative model probabilities indicated very strong evidence for MDFT were more prone to context effects than the remaining 35 participants, we contrasted the mean *RST*s of the two groups. Results indicated that these 13 participants had higher *RST*s for the compromise triplets (68% vs. 53%, HDI_{95} .02–.26, mean: .14, 99% of the HDI > 0), but not for the attraction triplets (64% vs. 63%, HDI_{95} -.05–.08, mean: .02, 70% of the HDI > 0) or the similarity triplets (54% vs. 54%, HDI_{95} -.12–.11, mean: .01, 46% of the HDI > 0).

As an alternative model selection criterion, we also compared the models using cross-validation (Browne, 2000; Stone, 1974). As a first step of this analysis, we split the choice data of each participant into two parts, a calibration sample to fit the model parameters and a validation sample to test the model predictions. To create the validation sample, we randomly drew two attraction, two compromise, and two similarity triplets. The remaining data were used for the calibration sample. To ensure that the results were not influenced by this random selection, we repeated this procedure four times. Next, we compared the models' log-likelihoods for both the validation and the calibration set, averaged across the four samples. In all four samples, participants' choices in the calibration set were best described by MDFT (μ_{LL} across samples = -52.86), followed by the probit model (μ_{LL} across samples = -60.49) and the logit model (μ_{LL} across samples = -61.27). More importantly, the results showed that MDFT also made the most accurate predictions for the validation sample (μ_{LL} across samples = -5.23), followed by the logit model (μ_{LL} across samples = -5.82), the probit model (μ_{LL} across samples = -5.90), and the baseline model (μ_{LL} across samples = -6.59). These results were also reproduced on the individual level, where most participants were most accurately predicted by MDFT (mean across samples: 68%), followed by the probit (mean across samples: 20%) and logit (mean across samples: 12%) models.

Besides allowing these quantitative model comparisons, the data on hand also provide the opportunity to test to what extent the models predict the observed correlations between the context effects. As a first step toward this analysis, we calculated the models' predicted probabilities for each participant of choosing the target and competitor, across all attraction, compromise, and similarity triplets. Next, we derived the *predicted relative choice share of the target (PRST)* for each participant according to

$$PRST = \text{mean}\left(\frac{probability\ \text{targets}}{probability\ \text{targets} + probability\ \text{competitors}}\right) \qquad (9)$$

In a third step, we calculated the correlations of the individual *PRST*s between the context effects, separately for each model. Figure 7 plots these *PRST*s for each model. As can be seen from the figure, only MDFT predicted a clear negative correlation between similarity and attraction triplets (logit model: r = -.09, SD = .01; probit model: r = -.14, SD = .01; MDFT: r = -.53, SD = .05) and both MDFT and the probit model accounted for the negative correlation between similarity and compromise triplets (logit model: r = -.09, SD = .01; probit model: r = -.38, SD = .03; MDFT: r = -.45, SD = .06). None of the models predicted a strong positive correlation between attraction and compromise triplets (logit model: r = .12, SD = .01; probit model: r = .17, SD = .01; MDFT: r = .02, SD = .01).

Next, we compared the predicted *PRST* to the observed *RST* separately for the attraction, compromise, and similarity triplets. The results indicated high correlations for MDFT (attraction: r = .67, SD = .06; compromise: r = .81, SD = .12; similarity: r = .64, SD = .10), followed by the probit (attraction: r = .45, SD = .03; compromise: r = .48, SD = .06; similarity: r = .36, SD = .06), and logit (attraction: r = .49, SD = .04; compromise: r = .30, SD = .05; similarity: r = .35, SD = .06) models.

As in Study 1, we also compared the estimated attribute weights between the models. The correlations between attribute weights of MDFT and the probit model, between MDFT and the logit model, and between the probit and logit model ranged from .59 to .76, from .21 to .39, and from .19 to .34, respectively (see the Supplemental Materials for the ranges of the estimated parameters). Thus, attribute weights obtained through MDFT were more similar to the probit than to the logit model. In general, all correlations were lower than those observed in Study 1.

Discussion of Study 2

In Study 2 we successfully elicited multiple context effects within single individuals. In theory, this provides a case in which MDFT should have an advantage over standard RUMs for predicting choices because MDFT can account for multiple context effects. In line with this proposition, taking model complexity into account, the decisions of most participants were best described (according to the BIC) and predicted (assessed by cross-validation) by MDFT as compared to the logit and probit models.

Correlation between context effects. Eliciting the attraction, the compromise, and the similarity effect simultaneously in a within-subject design allowed us to test for possible correlations between the three context effects. Interestingly, we observed that the attraction and compromise effects were positively correlated and that both were negatively correlated with the similarity effect. These correlations also provide a conjecture as to why it is difficult to elicit all three context effects within a single individual: If both the attraction and the compromise effect are negatively correlated with the similarity effect, then eliciting one of the first two effects also makes finding the similarity effect less likely, and vice versa.

Our results also indicate that the compromise and similarity effect were not as strong as the attraction effect. We can think of two reasons for this difference. First, the added option for compromise choice triplets might not have been placed far enough from the other options in the attribute space to be perceived as an extreme option. If so, it could explain why the target was chosen less frequently in this condition. Second, as outlined above, due to rounding half of the similarity choice triplets became attraction choice triplets and had to be excluded from the analysis, which in turn reduced the statistical power to detect similarity effects. The only other studies investigating multiple context effects within the same person did not report the correlations between the effects, which makes it hard to assess the relative magnitude of the correlations we found (Trueblood, Brown, & Heathcote, 2013; Tsetsos et al., 2012).

Predicted correlations between context effects. The observed correlations are in accordance with the MDFT predictions that Roe and colleagues (2001) derived from a set of theoretically derived parameters. They predicted a negative correlation between the similarity effect with both the attraction and the compromise effect and a positive correlation between the attraction and the compromise effect, which provides further evidence for MDFT.

Based on the estimated models' parameters, only MDFT predicted a negative correlation between the attraction and the similarity effect. Both MDFT and the probit model predicted a negative correlation between the similarity and the compromise effect. These predictions might be misleading for the probit model. That is because even though the probit model cannot predict the single context effects (Busemeyer et al., 2007), it is nevertheless possible to find a correlation between context effects. For example, in the probit model the *PRST* for similarity and compromise choice triplets can be lower than .50 (i.e., indicating no context effect), but can still correlate for PRST values below .50. Besides, Figure 7 shows rather narrowly distributed *PRST* for the probit model as compared to the more widely distributed *PRST* for MDFT, which seems to be more in line with the observed *RST* (Figure 5). This provides further evidence for MDFT.

General Discussion

The present work followed the idea that people make choices by comparing options with each other and thereby violate the principle of independent evaluations of options. To explain these interdependent evaluations, different theories have been proposed in the past literature including the prominent multi-alternative decision field theory (Roe et al., 2001). The goal of our work was to show that MDFT (Roe et al., 2001) provides an accurate empirical description of interdependent preferences and thereby outcompetes standard RUMs, such as the logit and probit models. Even though these simple RUMs have been repeatedly criticized in the past (Busemeyer et al., 2007; Rieskamp et al., 2006), they still represent the standard approach for predicting choice behavior in economics, psychology, consumer research, and related fields (e.g., Train, 2003). At the same time, cognitive process models, such as MDFT, have been called for (e.g., Otter et al., 2008). Therefore an empirical test of MDFT against the RUMs appeared necessary.

To make MDFT testable based on empirical grounds, we utilized a generalized distance function that yields the similarities between options within a choice set described on different attributes weights (Berkowitsch et al., in press). In addition to that, we also assumed that a decision is made when the preference state that develops over time converges to a constant value. Making this simplifying assumption reduces the computational effort for calculating the model's predicted choice probabilities for a large set of choice situations as it allows for closed-form representations of the model. However, with this simplification of the decision process MDFT loses its ability to predict decision times.

Study 1 confirmed that MDFT can be successfully fitted to observed choice data, but in the context on hand it did not necessarily outperform the more parsimonious logit and probit models when model complexity is taken into account. Thus, when the goal is to predict the outcome in simple preferential choice situations such as those in Study 1, applying RUMs seems justified on pragmatic grounds, as they can be easily implemented and estimated.

In Study 2, which used a within-subject experimental design, we showed that MDFT outperformed RUMs in situations where people’s choices were systematically influenced by the context in which the options were presented. Presumably, this is because MDFT can account for these context effects, whereas RUMs assume that options are evaluated independently from each other. Note that nevertheless a quarter of the participants were still assigned to the logit model, suggesting that not all participants were sensitive to context effects. The experimental design of Study 2 further provided the opportunity to explore the correlation between different context effects. Toward a better understanding of these correlations, choice models need to be able to account for multiple context effects simultaneously and to describe how they emerge. Cognitive process models such as MDFT depict promising theories to predict and explain preferential choices and their underlying evaluation processes.

When creating the choice set for Study 1, we excluded choice triplets with dominant options. Therefore, we did not expect attraction effects to occur in this study. Other than that, the choice tasks were quite similar between the two studies and thus we have no reason to believe that the cognitive processes that participants utilized were very different between the studies. Thus, the fact that the logit and the probit models fitted the data equally well as MDFT in Study 1 was probably due to the way we selected the choice options. However, to clearly show that the cognitive process underlying the choices in Study 1 were the same as in Study 2 requires further data that go beyond the scope of our experiment.

Advantages of Process Models

Although past research yields a considerable improvement of the simple RUMs so that some of their variants can account for systematic context effects (e.g., Kamenica, 2008; Kivetz, Netzer, & Srinivasan, 2004a, 2004b; Orhun, 2009; Rooderkerk, van Heerde, & Bijmolt, 2011), these models mostly remain silent about the cognitive process underlying decision making. At the same time, a growing body of literature inside and outside psychology promotes the application of cognitive process models to better understand and predict choice behavior (e.g., Chandukala et al., 2007; Otter et al. 2008; Reutskaja et al., 2011).

In the present work, we focused on MDFT as one prominent sequential sampling model that applies to preferential choices. However, other sequential sampling models, such as the leaky competing accumulator model (LCA; Usher & McClelland, 2004) or the multi-attribute linear ballistic accumulator model (MLBA; Trueblood, Brown, & Heathcote, 2013) can also account for context effects. The LCA (Usher & McClelland, 2004) in theory also predicts a positive correlation between the attraction and compromise effect, as the same mechanism (i.e., loss aversion) is responsible for producing the two effects (see also Tsetsos et al., 2010). Because the similarity effect is highest when loss aversion is absent, the LCA predicts negative correlations with the other two effects, which are also in line with our observations. We did not include a test of LCA or MLBA in comparison to MDFT, because as psychological process models they are conceptually similar and they are all in contrast to standard economic models that just focus on observed outcomes.

In comparison to standard economic models, cognitive process models have additional advantages. For instance, multiple studies have shown that context effects vary over deliberation time (Dhar, Nowlis, & Sherman, 2000; Lin, Sun, Chuang, & Su, 2008; Pettibone, 2012). These findings strengthen the relevance of process models, such as MDFT and LCA. Recently, these models have also been linked to neurological processes in the brain (e.g., Forstmann et al., 2010; Gluth, Rieskamp, & Büchel, 2012, 2013). Despite these advantages, so far comparisons between these models, for example, between MDFT and LCA, have mainly relied on theoretical arguments (Pettibone, 2012). Presumably, this is the case because it has proven somewhat difficult to actually fit these models to empirical data.

To advance our understanding of the cognitive processes that govern preferential choices, it is important to compare these models on empirical grounds. Toward this goal, providing empirically testable versions of these models also allows putting them to practical use, for example, as a feasible replacement for RUMs that, despite their limitations, still represent the standard approach in many applied fields such as market research. Besides, unlike with RUMs, the application of cognitive process models is not limited to preferential choice tasks; they have also been successfully applied to perceptual (Trueblood, Brown, Heathcote, & Busemeyer, 2013), inferential (Trueblood, 2012), and risky choice tasks (Tsetsos et al., 2012).

Models like MDFT and the logit and probit models are not the end points of the complexity continuum as more complex as well as simpler models might exist. For example, Payne, Bettmann, and Johnson (1993) provide an overview of different strategies people could follow for making preferential choices, such as a simple lexicographic heuristic that focuses only on one single attribute. On the other hand, we applied a simplified version of MDFT by assuming infinite decision time, $t \rightarrow \infty$. That means the fully specified version of MDFT (Roe et al., 2001) is more complex. The MLBA as an alternative sequential sampling model cannot easily be placed on this (one-dimensional) complexity continuum, as it additionally provides response time data. A different approach to compare the models' complexities is to apply a Bayesian approach that weights the model's possible predictions by its prior probability and additionally accounts for the functional form of the model parameters (e.g., Scheibehenne, Rieskamp, & Wagenmakers, 2013). As the attraction effect was particularly strong compared to the compromise and similarity effect, simpler models that only predict the attraction effect might have yielded comparable model fit to MDFT with less complexity.

Practical and Policy Implications

One of the reasons for the widespread use of RUMs such as the logit and probit models probably is their relative ease of implementation. Although the advantages of cognitive process models have been acknowledged, in applied contexts—such as market research—these models are rarely applied because so far, several model specifications have remained unclear (Otter et al., 2008). As our results show, cognitive process models can be readily applied to actual empirical data indicating that they can provide a valuable alternative to RUMs to predict people's preferences. In particular, the present work specified MDFT such that its parameters could be estimated from empirical data and choice behavior could be predicted. Because this specification came at the price of losing the ability to make predictions about decision time, future developments of MDFT should aim at further modifying the model so that all parameters can be estimated.

In an applied setting, researchers might also be interested in the subjective importance weights that people assign to specific attributes. For instance, in a consumer context marketing companies want to infer the weights given to products' attributes. In the medical domain, physicians may want to know the importance people attach to the different aspects of a treatment, such as the treatment's success as compared to its side effects. In the educational domain, it is crucial to know how much importance teachers, parents, and pupils give to the topics taught at school. A feasible way to investigate these questions is by conducting choice studies and applying RUMs, such as the logit or probit model, to infer the importance of different aspects. However, given that decisions are systematically influenced by context effects, the estimated importance weights may be biased. In Study 1, MDFT and the RUMs highly agreed on the estimated attribute weights, providing evidence that they could be used to elicit the importance the participants gave to the different attributes. However, this agreement was much lower in Study 2. Here, the RUMs performed worse than MDFT in predicting the observed choices, so the estimated attribute weights of the RUMs in Study 2 may not necessarily reflect participants' "true" importance weights (for a discussion on estimating importance weights, see Marley, Flynn, & Louviere, 2008, and Marley & Pihlens, 2012).

Technically, RUMs can to some extent account for context effects by adjusting the attribute weights, which may lead to unreliable and biased results. At the same time, the interdependent evaluations of choice options and hence the occurrence of context effects is widespread: For instance, there is a large body of research on the influence of context effects on hiring decisions (Aaker, 1991; Highhouse, 1996, 1997; Slaughter, 2007; Slaughter, Bagger, & Li, 2006; Slaughter & Highhouse, 2003; Slaughter, Kausel, & Quiñones, 2011; Slaughter, Sinar, & Highhouse, 1999; Tenbrunsel & Diekmann, 2002). Likewise, context effects have been reported for perceptual (Trueblood, Brown, Heathcote, & Busemeyer, 2013), inferential (Trueblood, 2012), and risky choice tasks (Tsetsos et al., 2012). For a meta-analysis of context effects for various consumer products, see Heath and Chatterjee (1995). Together, these studies illustrate that the subjective importance weights and the predicted choice proportions by RUMs might be less reliable when context effects are likely to occur. Our results indicate that it is in these situations where the application of cognitive models such as MDFT is probably most advantageous.

References

Aaker, J. (1991). The negative attraction effect? A study of the attraction effect under judgment and choice. *Advances in Consumer Research, 18*, 462-469.

Adamowicz, W., Louviere, J., & Williams, M. (1994). Combining revealed and stated preference methods for valuing environmental amenities. *Journal of Environmental Economics and Management*, *26*, 271-292.

Attneave, F. (1950). Dimensions of similarity. *American Journal of Psychology, 63*, 516-556.

Berkowitsch, N. A. J., Scheibehenne, B., Matthäus, M., & Rieskamp, J. (2013). *Determining the similarity between multiattribute options: A generalized similarity function*. Manuscript submitted for publication.

Bowler, S., Karp, J. A., & Donovan, T. (2010). Strategic coalition voting: Evidence from New Zealand. *Electoral Studies*, *29*, 350-357. doi: 10.1016./j.electstud.2010.03.001

Browne, M. W. (2000). Cross-validation methods. *Journal of Mathematical Psychology, 44*, 108-132. doi: 10.1006/jmps.1999.1289

Busemeyer, J. R., Barkan, R., Mehta, S., & Chaturvedi, A. (2007). Context effects and models of preferential choice: Implications for consumer behavior. *Marketing Theory, 7*, 39-58. doi: 10.1177/1470593107073844

Busemeyer, J. R., & Diederich, A. (2002). Survey of decision field theory. *Mathematical Social Sciences, 43*, 345-370. doi: 10.1016/S0165-4896(02)00016-1

Busemeyer, J. R., & Townsend, J. T. (1993). Decision field theory: A dynamic-cognitive approach to decision making in an uncertain environment. *Psychological Review, 100*, 432-459.

Busemeyer, J. R., & Wang, Y. (2000). Model comparisons and model selections based on generalization criterion methodology. *Journal of Mathematical Psychology, 44*, 171-189. doi: 10.1006jmps.1999.1282

Carmon, Z., & Simonson, I. (1998). Price-quality trade-offs in choice versus matching: New insights into the prominence effect. *Journal of Consumer Psychology, 7*, 323-343.

Chandukala, S. R., Kim, J., Otter, T., Rossi, P. E., & Allenby, G. M. (2007). Choice models in Marketing: Economic assumptions, challenges and trends. *Foundations and Trends in Marketing, 2*, 97-184. doi: 10.1561/1700000008

Daganzo, C. (1980). *Multinomial probit*. New York, NY: Academic Press.

Dhar, R., Nowlis, S. M., & Sherman, S. J. (2000). Trying hard or hardly trying: An analysis of context effects in choice. *Journal of Consumer Psychology, 9*, 189-200. doi: 10.1207/S15327663JCP0904_1

Diederich, A. (2010). Mathematical Psychology. In: Irving B. Weiner & W. E. Craighead (Eds.) *The Corsini Encyclopedia of Psychology* (4th ed., pp. 971-974). New York, NY: Wiley.

Forstmann, B. U., Anwander, A., Schäfer, A., Neumann, J., Brown, S., Wagenmakers, E.-J., … Turner, R. (2010). Cortico-striatal connections predict control over speed and accuracy in perceptual decision making. *Proceedings of the National Academy of Sciences, 107*, 15916-15920. doi: 10.1073/pnas.1004932107

Gil, J. M., & Sánchez, M. (1997). Consumer preferences for wine attributes: A conjoint approach. *British Food Journal*, *99*, 3-11.

Gluth, S., Rieskamp, J., & Büchel, C. (2012). Deciding when to decide: Time-variant sequential sampling models explain the emergence of value-based decisions in the human brain. *Journal of Neuroscience, 32*, 10686-10698. doi:10.1523/JNEUROSCI.0727-12.2012

Gluth, S., Rieskamp, J., & Büchel, C. (2013). Classic EEG motor potentials track the emergence of value-based decisions. *NeuroImage, 79*, 394-403. doi: 10.1016/j.neuroimage.2013.05.005

Green, P. E., & Srinivasan, V. (1978). Conjoint analysis in consumer research: Issues and Outlook. *Journal of Consumer Research*, *5*, 103-123.

Green, P. E., & Srinivasan, V. (1990). Conjoint analysis in marketing: New developments with implications for research and practice. *Journal of Marketing*, *54*, 3-19.

Hanley, N., Wright, R. E., & Adamowicz, V. (1998). Using choice experiments to value the environment. *Environmental and Resource Economics*, *11*, 413-428.

Hausman, J. A., & Wiese, D. A. (1978). A conditional probit model for qualitative choice: Discrete decisions recognizing interdependence and heterogeneous preferences. *Econometrica, 46*, 403-426.

Heath, T. B., & Chatterjee, S. (1995). Asymmetric decoy effects on lower-quality versus higher-quality brands: Meta analytic and experimental evidence. *Journal of Consumer Research, 22*, 268-284.

Hensher, D. A. (1994). Stated preference analysis of travel choices: The state of practice. *Transportation*, *21*, 107-133.

Highhouse, S. (1996). Context-dependent selection: The effects of decoy and phantom job candidates. *Organizational Behavior and Human Decision Processes*, *65*, 68-76. doi: 10.1006/obhd.1996.0006

Highhouse, S. (1997). Understanding and improving job-finalist choice: The relevance of behavioral decision research. *Human Resource Management Review, 7*, 449-470. doi: 10.1016/S1053-4822(97)90029-2

Hotaling, J. M., Busemeyer, J. R., & Li, J. (2010). Theoretical developments in decision field theory: Comment on Tsetsos, Usher, and Chater (2010). *Psychological Review, 117*, 1294-1298. doi: 10.1037/a0020401

Huber, J., Payne, J. W., & Puto, C. (1982). Adding asymmetrically dominated alternatives: Violations of regularity and the similarity hypothesis. *Journal of Consumer Research, 9*, 90-98.

Huber, J., & Puto, C. (1983). Market boundaries and product choice: Illustrating attraction and substitution effects. *Journal of Consumer Research, 10*, 31-44.

Johnson, J. G., & Busemeyer, J. R. (2010). Decision making under risk and uncertainty. *Wiley Interdisciplinary Reviews: Cognitive Science, 1*, 736-749. doi: 10.1002/wcs.76

Kamenica, E. (2008). Contextual inferences in markets: On the informational content of product lines. *American Economic Review, 98*, 2127-2149.

Karp, J. A. (2009). Candidate effects and spill-over in mixed systems: Evidence from New Zealand. *Electoral Studies*, *28*, 41-50. doi: 10.1016/j.electstud.2008.06.012

Kivetz, R., Netzer, O., & Srinivasan, V. (2004a). Models for capturing the compromise effect. *Journal of Marketing Research, 41*, 237-257.

Kivetz, R., Netzer, O., & Srinivasan, V. (2004b). Extending compromise effect models to complex buying situations and other context effects. *Journal of Marketing Research, 41*, 262-268.

Koistinen, L., Pouta, E., Heikkilä, J., Forsman-Hugg, S., Kotro, J., Mäkelä, J., & Niva, M. (2013). The impact of fat content, production methods and carbon footprint information on consumer preferences for minced meat. *Food Quality and Preference, 29*, 126-136. doi: 10.1016/j.foodqual.2013.03.007

Kruschke, J. K. (2011a). Bayesian assessment of null values via parameter estimation and model comparison. *Perspectives on Psychological Science, 6*, 299-312. doi: 10.1177/1745691611406925

Kruschke, J. K. (2011b). *Doing Bayesian data analysis*. *A tutorial with R and BUGS*. Burlington, MA: Academic Press.

Lewandowsky, S., & Farrell, S. (2011). *Computational modeling in cognition*. Thousand Oaks, CA: Sage.

Lin, C.-H., Sun, Y.-C., Chuang, S-C., & Su, H.-J. (2008). Time pressure and the compromise and attraction effects in choice. *Advances in Consumer Research, 35*, 348-352.

Loureiro, M. L., & Umberger W. J. (2005). Assessing consumer preferences for country-of-origin labeling. *Journal of Agricultural and Applied Economics*, *37*, 49-63.

Loureiro, M. L., & Umberger W. J. (2007). A choice experiment model for beef: What US consumer responses tell us about relative preferences for food safety, country-of-origin labeling and traceability. *Food Policy*, *32*, 496-514. doi: 10.1016/j.foodpol.2006.11.006

Luce, R. D. (1959). *Individual choice behavior*. Oxford, England: Wiley.

Marley, A. A. J., Flynn, T. N., & Louviere, J. J. (2008). Probabilistic models of set-dependent and attribute-level best-worst choice. *Journal of Mathematical Psychology, 52*, 283-298. doi: 10.1016/j.jmp.2008.02.002

Marley, A. A. J. & Pihlens, D. (2012). Models of best-worst choice and ranking among multiattribute options (profiles). *Journal of Mathematical Psychology, 56*, 24-34. doi: 10.1016/j.jmp.2011.09.001

McClelland, J. L., & Rumelhart, D. E. (1981). An interactive acitivation model of context effects in letter perception: Part 1. An account of basic findings. *Psychological Review, 88*, 375-407.

McFadden, D. (1973). Conditional logit analysis of qualitative choice behavior. In P. Zarembka (Ed.), *Frontiers in econometrics* (pp. 105-142). New York, NY: Academic Press.

McFadden, D. (2001). Economic choices. *American Economic Review, 91*, 351-378.

Menard, S. (2004). Six approaches to calculating standardized logistic regression coefficients. *American Statistician, 58*, 218-223. doi:10.1198/000313004X946

Mosteller, F., & Nogee, P. (1951). An experimental measurement of utility. *Journal of Political Economy, 59*, 371-404.

Nownes, A. J. (1992). Primaries, general elections, and voter turnout: A multinomial logit model of the decision to vote. *American Politics Research*, *20*, 205-226. doi: 10.1177/1532673X9202000204

Orhun, A. Y. (2009). Optimal product line design when consumers exhibit choice-set dependent preferences. *Marketing Science, 28*, 868-886.

Otter, T., Johnson, J., Rieskamp, J., Allenby, G. M., Brazell, J. D., Diederich, A., … Townsend, J. (2008). Sequential sampling models of choice: Some recent advances. *Marketing Letters, 19*, 255-267. doi: 10.1007/s11002-008-9039-0

Payne, J. W., Bettman, J. R., & Johnson, E. (1993). *The adaptive decision maker*. Cambridge, England: Cambridge University Press.

Pettibone, J. C. (2012). Testing the effect of time pressure on asymmetric dominance and compromise decoys in choice. *Judgment and Decision Making, 7*, 513-523.

Phillips, K. A., Maddala, T., & Johnson, F. R. (2002). Measuring preferences for health care interventions using conjoint analysis: An application to HIV testing. *Health Services Researches*, *37*, 1681-1705.

Raftery, A. E. (1995). Bayesian model selection in social research. *Sociological Methodology, 25*, 111-163.

Ratcliff, R. (1978). A theory of memory retrieval. *Psychological Review, 85*, 59-108. doi: 10.1037/0033-295X.85.2.59

Reutskaja, E., Nagel, R., Camerer, C. F., & Rangel, A. (2011). Search dynamics in consumer choice under time pressure: An eye-tracking study. *American Economic Review, 101*, 900-926.

Rieskamp, J. (2008). The probabilistic nature of preferential choice. *Journal of Experimental Psychology, Learning, Memory, and Cognition, 34*, 1446-1456. doi: 0.1037/a0013646

Rieskamp, J., Busemeyer, J. R., & Mellers, B. A. (2006). Extending the bounds of rationality: Evidence and theories of preferential choice. *Journal of Economic Literature, 44*, 631-661.

Roberts, D. C., Boyer, T. A., & Lusk, J. L. (2008). Preferences for environmental quality under uncertainty. *Ecological Economics*, *66*, 584-593. doi: 10.1016/j.ecolecon.2008.05.010

Roe, R. M., Busemeyer, J. R., & Townsend, J. T. (2001). Multialternative decision field theory: A dynamic connectionist model of decision making. *Psychological Review, 108*, 370-392. doi: 10.1037//0033-295X.108.2.370

Rooderkerk, R. P., van Heerde, H. J., & Bijmolt, T. H. A. (2011). Incorporating context effects into a choice model. *Journal of Marketing Research, 48*, 767-780. doi: 10.1509/jmkr.48.4.767

Ryan, M., & Farrar, S. (2000). Using conjoint analysis to elicit preferences for health care. *British Medical Journal*, *320*, 1530-1533.

Scheibehenne, B., Rieskamp, J., & Gonzalez-Vallejo, C. (2009). Cognitive models of choice: Comparing decision field theory to the proportional difference model. *Cognitive Science, 33*, 911-939. doi: 10.1111/j.1551-6709.2009.01034.x

Scheibehenne, B., Rieskamp, J, & Wagenmakers, E.-J. (2013). Testing adaptive toolbox models: A Bayesian hierarchical approach. *Psychological Review, 120*, 39-64. doi: 10.1037/a0030777

Simonson, I., & Tversky, A. (1992). Choice in context: Tradeoff contrast and extremeness aversion. *Journal of Marketing Research, 24*, 281-295.

Slaughter, J. E. (2007). Effects of two selection batteries on decoy effects in job-finalist choice. *Journal of Applied Social Psychology, 37*, 76-90. doi: 10.1111/j.0021-9029.2007.00148.x

Slaughter, J. E., Bagger, J., & Li, A. (2006). Context effects on group-based employee selection decisions. *Organizational Behavior and Human Decision Processes, 100*, 47-59. doi: 10.1016/j.obhdp.2006.01.003

Slaughter, J. E., & Highhouse, S. (2003). Does matching up features mess up job choice? Boundary conditions on attribute-salience effects. *Journal of Behavioral Decision Making, 16*, 1-15. doi: 10.1002/bdm.428

Slaughter, J. E., Kausel, E. E., & Quiñones, M. A. (2011). The decoy effect as a covert influence tactic. *Journal of Behavioral Decision Making, 24*, 249-266. doi: 10.1002/bdm.687

Slaughter, J. E., Sinar, E. F., & Highhouse, S. (1999). Decoy effects and attribute-level inferences. *Journal of Applied Psychology, 84,* 823-828. doi: 10.1037/0021-9010.84.5.823

Slovic, P., & Tversky, A. (1974). Who accepts Savage's axiom? *Behavioral Science, 19*, 368.

Soltani, A., De Martino, B., & Camerer, C. (2012). A range-normalization model of context-dependent choice: A new model and evidence. *PLOS Computational Biology, 8*, 1-15.

Stone, M. (1974). Cross-validatory choice and assessment of statistical predictions. *Journal of the Royal Statistical Society, 36B*, 111-147.

Tenbrunsel, A. E., & Diekmann, K. A. (2002). Job decision inconsistencies involving social comparison information: The role of dominating alternatives. *Journal of Applied Psychology, 87*, 1149-1158. doi: 10.1037//0021-9010.87.6.1149

Thurstone, L. L. (1927). A law of comparative judgment. *Psychological Review, 101*, 266-270.

Townsend, J. T., & Ashby, F. G. (1983). *Stochastic modeling of elementary psychological processes.* Cambridge, England: Cambridge University Press.

Train, K. (1978). A validation test of a disaggregate mode choice model. *Transportation Research*, *12*, 167-174.

Train, K. E. (2003). *Discrete choice methods with simulation*. Cambridge, England: Cambridge University Press.

Trueblood, J. S. (2012). Multialternative context effects obtained using an inference task. *Psychonomic Bulletin & Review, 19*, 9962-9968. doi: 10.3758/s13423-012-0288-9

Trueblood, J. S., Brown, S. D., Heathcote, A. (2013). The multi-attribute linear ballistic accumulator model of decision-making. In M. Knauff, M. Pauen, N. Sebanz, & I. Wachsmuth (Eds.), *Proceeding of the 35th Annual Conference of the Cognitive Science Society.* Austin, TX: Cognitive Science Society.

Trueblood, J. S., Brown, S. D., Heathcote, A., & Busemeyer, J. R. (2013). Not just for consumers: Context effects are fundamental to decision making. *Psychological Science, 24*, 901-908. doi: 10.1177/0956797612464241

Tsetsos, K., Chater, N., & Usher, M. (2012). Salience driven value integration explains decision biases and preference reversals. *Proceedings of the National Academy of Sciences, 109,* 9659-9664. doi: 10.1073/pnas.1119569109

Tsetsos, K., Usher, M., & Chater, N. (2010). Preference reversal in multiattribute choice. *Psychological Review, 117*, 1275-1293. doi: 10.1037/a0020580

Tversky, A. (1972a). Choice by elimination. *Journal of Mathematical Psychology, 9*, 341-367.

Tversky, A. (1972b). Elimination by aspects: A theory of choice. *Psychological Review, 79*, 281-299.

Tversky, A., Sattah, S., & Slovic, P. (1988). Contingent weighting in judgment and choice. *Psychological Review, 95*, 371-384.

Tversky, A., & Simonson, I. (1993). Context-dependent preferences. *Management Science, 39*, 1179-1189.

Usher, M., & McClelland, J. L. (2004). Loss aversion and inhibition in dynamical models of multialternative choice. *Psychological Review, 111*, 757-769.

Vickers, D. (1970). Evidence for an accumulator of psychophysical discrimination. *Ergonomics, 13*, 37-58.

von Neumann, J., & Morgenstern, O. (1947). *Theory of games and economic behavior* (2nd ed.). Princeton, NJ: Princeton University Press.

Wardman, M. (1988). A comparison of revealed preference and stated preference models of travel behavior. *Journal of Transport Economics and Policy*, *22*, 71-91.

Wedell, D. H. (1991). Distinguishing among models of contextually induced preference reversals. *Journal of Experimental Psychology, Learning, Memory, and Cognition, 7*, 767-778.

Footnotes

[1] Technically, this requires the eigenvalues of the feedback matrix to be smaller than 1 (Busemeyer & Diederich, 2002).

[2] Prior to calculating the correlations, the attribute weights of the logit model were standardized with respect to their standard deviations (Menard, 2004).

[3] Only correctly matched product pairs (i.e., the missing price of a *lighter* bike must be filled in as *more expensive* than the price of a heavier bike) were presented in the subsequent choice task.

Table 1

Products and Attributes Presented in Study 2

Product	Attribute 1	Attribute 2
Color printer	Printing speed in pages per minute	Price in Swiss francs
Digital camera	Picture quality in megapixels	Memory space in gigabytes
Notebook computer	Weight in kilograms	Battery longevity in hours
Racing bike	Weight in kilograms	Price in Swiss francs
Vacuum cleaner	Suction power in watts	Price in Swiss francs
Washing machine	Water consumption in liters	Life cycle in years

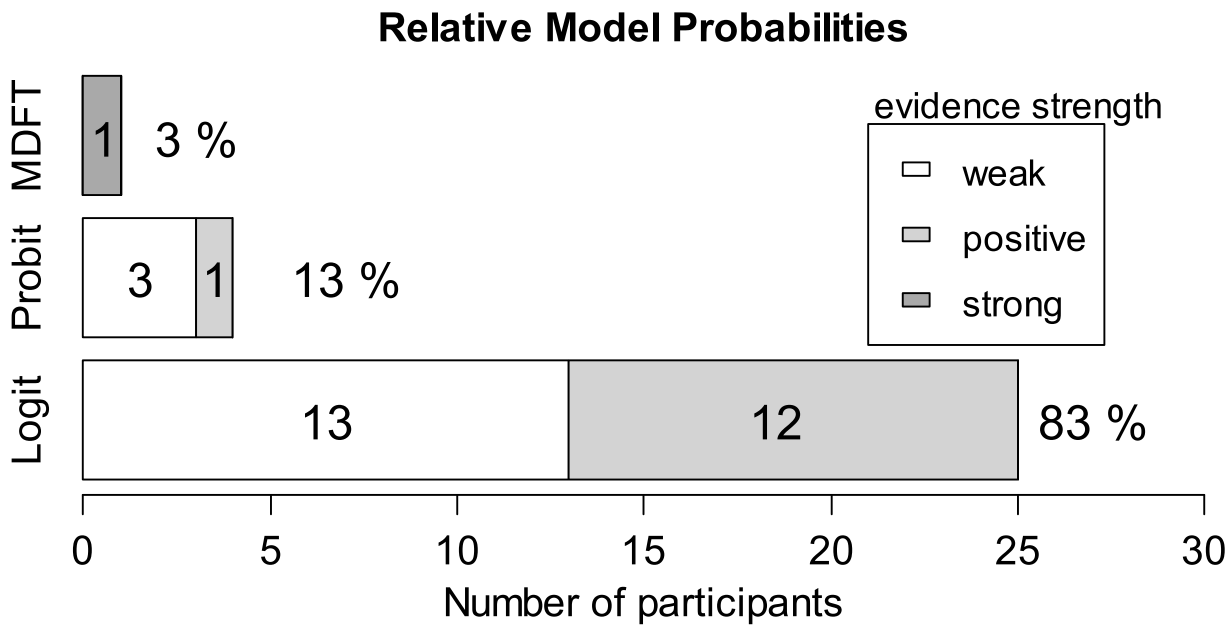

Figure 1. Model comparison of the relative model probabilities based on their Bayesian information criteria. Numbers in the bars indicate how many individuals fell into a specific level of evidence strength. The percentages indicate what proportions of participants were assigned to each model.

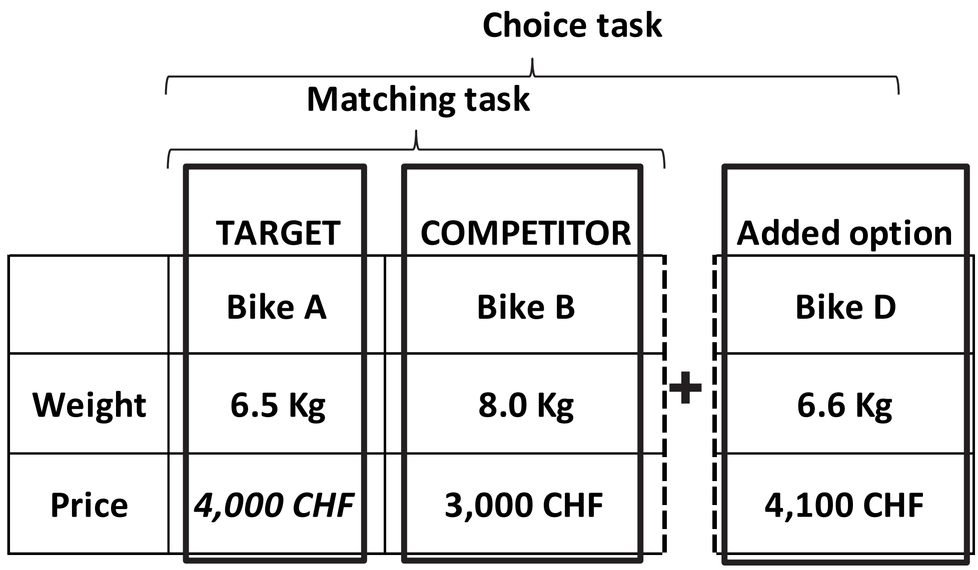

Figure 2. Example of a matching and a choice task for racing bikes. The price of Bike A was blank before participants matched the italic number (e.g., 4,000). Here, the choice task is intended to elicit an attraction effect (see also choice triplet ABD in Figure 3a).

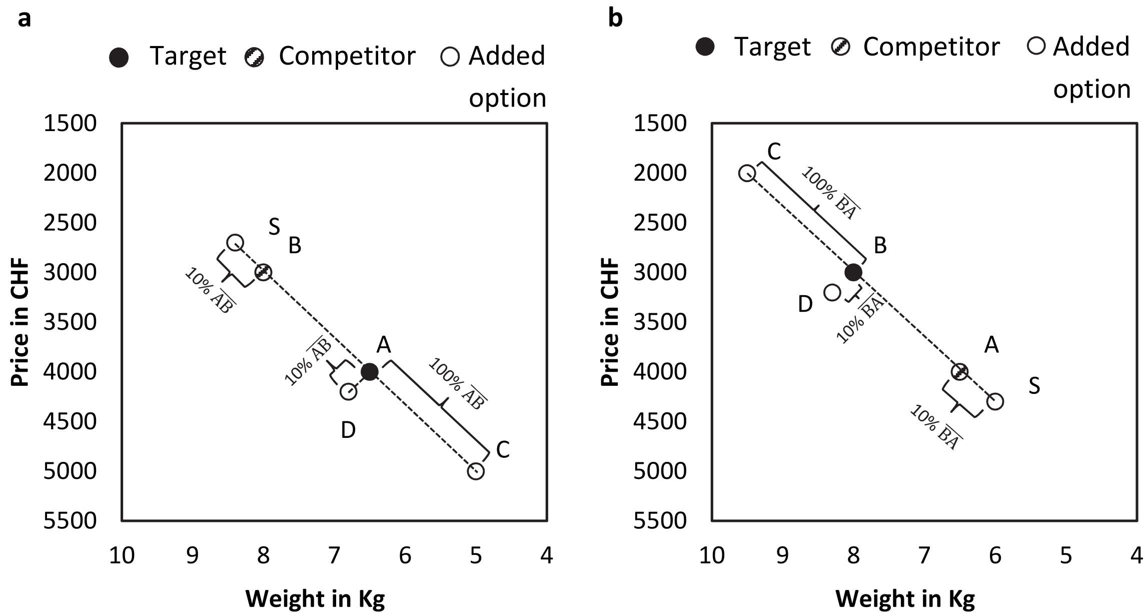

Figure 3. Illustrative example of the choice task, where either Option A (a) or Option B (b) is the target, depending on the position of the added option. For each participant, two matched options A and B were presented with one of the individually calculated decoys C, D, or S.

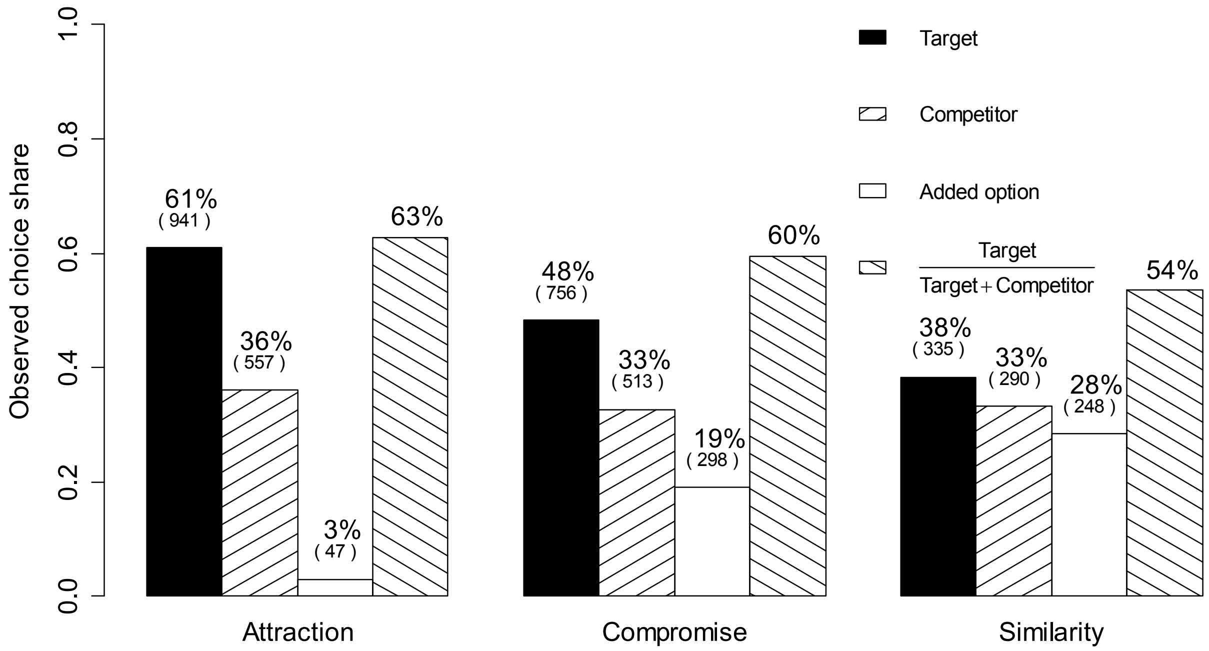

Figure 4. Observed choice shares across all participants' choices for the target, the competitor, the added option, and the relative choice shares of the target for the attraction, compromise, and similarity choice triplets. The percentages of the first three columns within each category add up to 100% except for the similarity choice triplets (due to rounding). Absolute numbers are shown in parentheses.

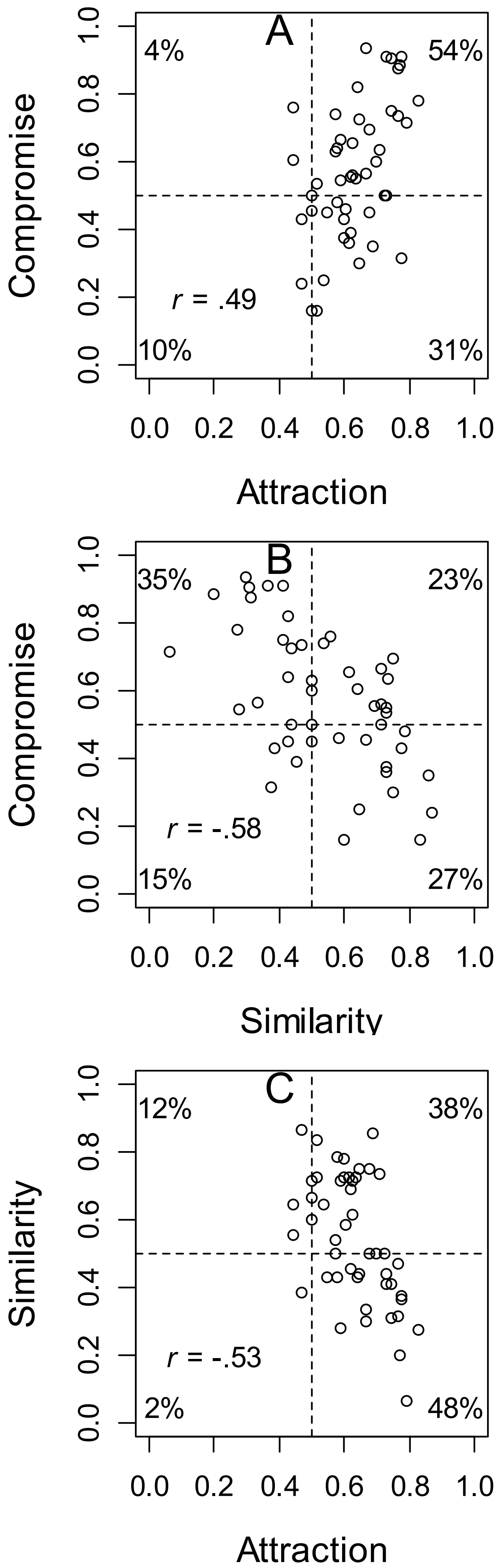
A
4%
54%
r = .49
10%
31%
Compromise
Attraction
0.0 0.2 0.4 0.6 0.8 1.0
B
35%
23%
r = -.58
15%
27%
Compromise
Similarity
0.0 0.2 0.4 0.6 0.8 1.0
C
12%
38%
r = -.53
2%
48%
Similarity
Attraction
0.0 0.2 0.4 0.6 0.8 1.0

Figure 5. Plot of the relative choice shares of the target for each participant. Within each panel, dots in the upper right quadrant indicate participants who are prone to both context effects, and dots in the upper left and lower right indicate participants who are prone to only one context effect. Dots in the lower left indicate participants who are not prone to either of the two context effects.

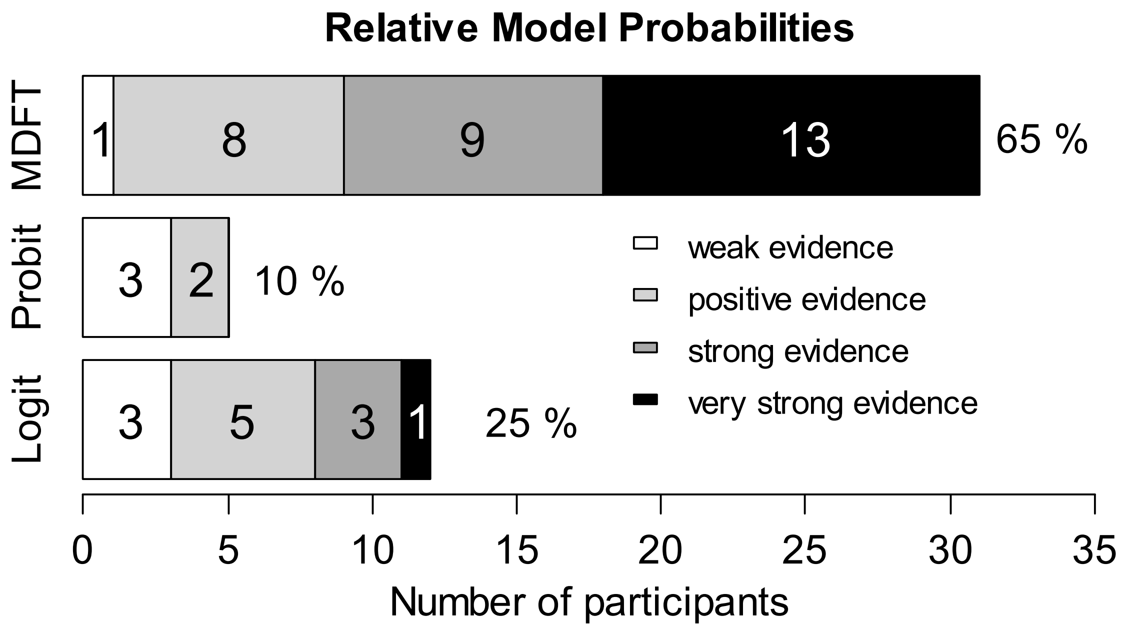

Figure 6. Model comparison of the relative model probabilities based on their Bayesian information criteria. Numbers in the bars indicate how many individuals fell into a specific level of evidence strength. The percentages indicate what proportions of participants were assigned to each model.

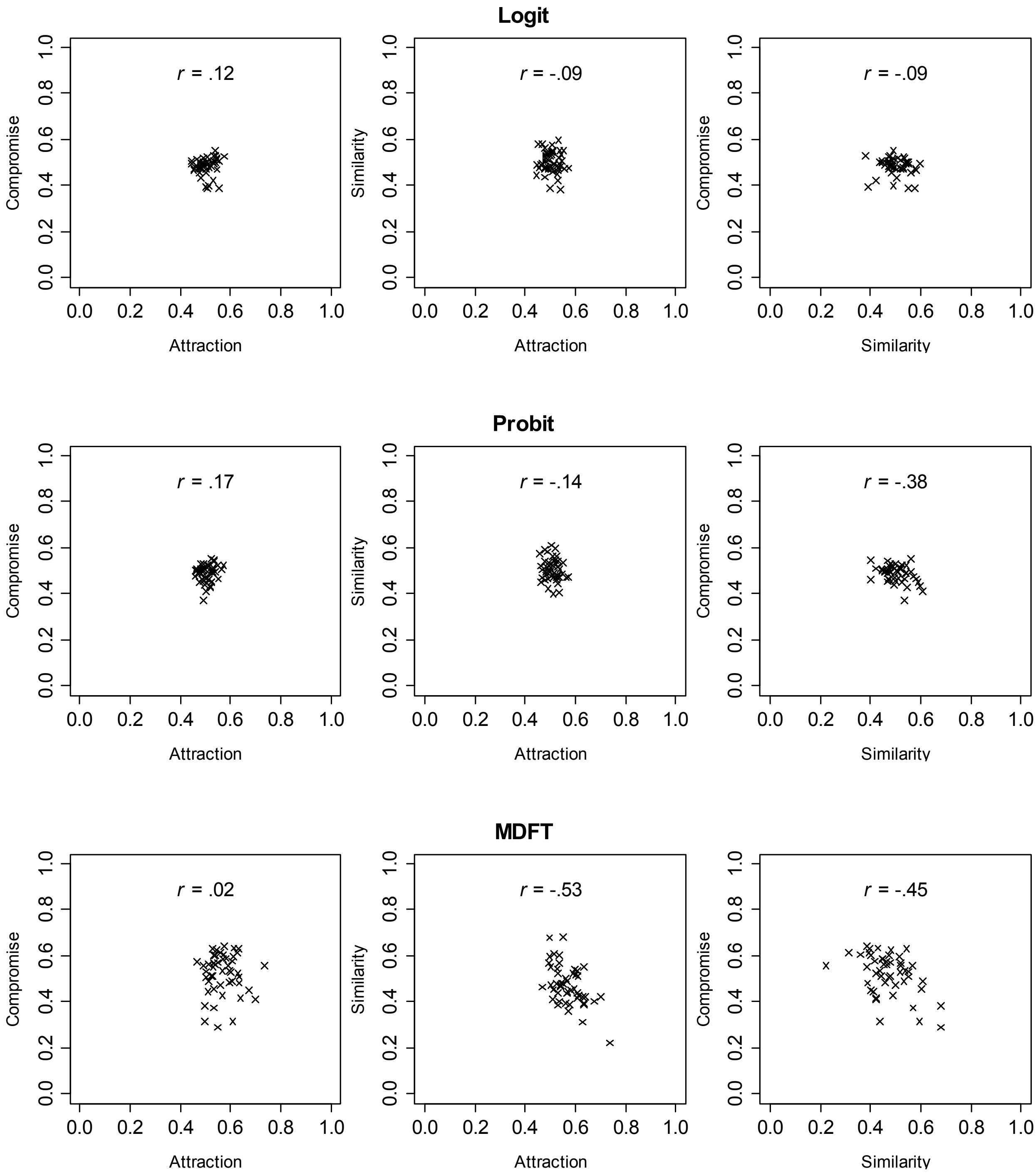

Figure 7. Comparison of the predicted relative choice shares of the target on an individual level across models and context effect.

Appendix A: Simplifying the Stopping Rule

In its original form, MDFT provides two explanations of how a decision is reached: Either an *internal stopping rule* leads to a decision or an external stopping rule limits the decision process. The internal stopping rule assumes that a decision is made as soon as the accumulated preference state for one option reaches a threshold θ. However, if a too high θ is set, the threshold might never be reached, since the preference states might have converged to a stable value before reaching θ. On the other hand, setting θ too low can lead to preference states that have not yet converged, and further evidence would potentially yield different choice probabilities. In theory, one could keep track of the change in choice probabilities to stop the estimation process as soon as convergence is reached. This is computationally unsatisfying, though, because obtaining stable choice probabilities is cumbersome. As an alternative, one can assume an *external stopping rule* with unconstrained deliberation time (i.e., a very high t). However, the associated time-intensive simulations and the required computational effort to fit MDFT are still unsatisfying (Trueblood, Brown, & Heathcote, 2013), as the preference state **P** needs to be iterated until t is reached for every set of tested parameters. A better way to think of decisions with no time constraints is to set $t \rightarrow \infty$, which according to Roe et al. (2001) reduces the calculation of the mean preference state over time $\xi(t)$ to

$$\xi(\infty) = (\boldsymbol{I} - \boldsymbol{S})^{-1}\mu \tag{A1}$$

Thus, instead of iterating for a very long time to calculate $\xi(t)$ we can directly calculate $\xi(\infty)$. Deriving choice probabilities for $t \rightarrow \infty$ further requires the variance–covariance matrix of the preference state. Busemeyer and colleagues (2006) suggested a formula for $\boldsymbol{\Omega}(\infty)$; however, this solution is limited to cases where the variance–covariance matrix of **V** is a diagonal matrix, that is, $\boldsymbol{\Phi} = \varphi^2 \cdot \boldsymbol{I}$. In the next section, we develop a general formula to directly calculate $\mathbf{\Omega}(\infty)$ for k options. This avoids time-consuming calculations of the variance–covariance matrix for each time point and it leads to stable choice predictions. The variance–covariance matrix at time t is calculated as

$$\mathbf{\Omega}_t = \sum_{j=0}^{t-1} \mathbf{S}^j \mathit{\Phi} \cdot \left(\boldsymbol{S}^j\right)' . \tag{A2}$$

where $\mathit{\Phi}$ is the $k \times k$ variance–covariance matrix of $\mathbf{V}$ (see Roe et al., 2001, Appendix B). From complete induction follows

$$\mathbf{\Omega}_{t+1} = \boldsymbol{S} \cdot \mathbf{\Omega}_t \cdot \boldsymbol{S}' + \mathit{\Phi}. \tag{A3}$$

By combining Equations A2 and A3 we can calculate the change in the variance–covariance matrix after one iteration (i.e., $t + 1$) by

$$\mathbf{\Omega}_{t+1} - \mathbf{\Omega}_t = \boldsymbol{S} \cdot \mathbf{\Omega}_t \cdot \boldsymbol{S}' + \mathit{\Phi} - \mathbf{\Omega}_t . \tag{A4}$$

For the feedback matrix $\boldsymbol{S}$ with eigenvalues smaller than 1, the sequence of the matrices $\boldsymbol{S}^t$ for $t \to \infty$ converges to a zero matrix. We can therefore neglect the term $\boldsymbol{S}^t$ after a certain (high enough) number of iterations, say, t_0. That is,

$$\mathbf{\Omega}_{t_0+j} = \mathbf{\Omega}_{t_0} \ \forall j \geq 0 \tag{A5}$$

from which follows

$$0 = \mathbf{\Omega}_{t_0+1} - \mathbf{\Omega}_{t_0} = \boldsymbol{S} \cdot \mathbf{\Omega}_{t_0} \cdot \boldsymbol{S}' + \mathit{\Phi} - \mathbf{\Omega}_{t_0} . \tag{A6}$$

This means that the covariance matrix of the preference state has converged and $\mathbf{\Omega}_{t_0} = \mathbf{\Omega}(\infty)$. We can reorganize Equation A6 as

$$\mathbf{\Omega}(\infty) - \boldsymbol{S} \cdot \mathbf{\Omega}(\infty) \cdot \boldsymbol{S}' = \mathit{\Phi} . \tag{A7}$$

Now we solve Equation A7 for $\mathbf{\Omega}(\infty)$ to obtain a system of linear equations. This is achieved by the following steps. We first explicitly calculate $\boldsymbol{S} \cdot \mathbf{\Omega}(\infty) \cdot \boldsymbol{S}'$. We next transform the $k \times k$ matrices $\boldsymbol{S} \cdot \mathbf{\Omega}(\infty) \cdot \boldsymbol{S}'$ and $\mathbf{\Omega}(\infty)$ into the $k^2 \times 1$ vectors $\overline{\boldsymbol{S} \cdot \mathbf{\Omega}(\infty) \cdot \boldsymbol{S}'}$ and $\overline{\mathbf{\Omega}(\infty)}$, respectively, so we can search the $k^2 \times k^2$ matrix $\boldsymbol{Z}$, which multiplied by $\overline{\boldsymbol{\Omega}(\infty)}$ is equivalent to $\overline{\boldsymbol{S} \cdot \boldsymbol{\Omega}(\infty) \cdot \boldsymbol{S}'}$, so that

$$\overline{\boldsymbol{S} \cdot \boldsymbol{\Omega}(\infty) \cdot \boldsymbol{S}'} = \boldsymbol{Z} \cdot \overline{\boldsymbol{\Omega}(\infty)}. \tag{A8}$$

Restructuring Equation A7 according to Equation A8 leads to

$$\overline{\boldsymbol{\Omega}(\infty)} - \boldsymbol{Z} \cdot \overline{\boldsymbol{\Omega}(\infty)} = \overline{\boldsymbol{\Phi}} \tag{A9}$$

where the $k^2 \times 1$ vector $\overline{\Phi}$ is the $k \times k$ transformed matrix Φ. Equation A9 can be solved for $\overline{\boldsymbol{\Omega}(\infty)}$:

$$\overline{\boldsymbol{\Omega}(\infty)} = (\boldsymbol{I} - \boldsymbol{Z})^{-1} \cdot \overline{\boldsymbol{\Phi}}. \tag{A10}$$

Finally, we retransform the $k^2 \times 1$ vector $\overline{\boldsymbol{\Omega}(\infty)}$ back into the $k \times k$ matrix $\boldsymbol{\Omega}(\infty)$. Now we have an analytical solution for $\boldsymbol{\xi}(\infty)$ and for $\boldsymbol{\Omega}(\infty)$, so that the choice probabilities for $t \to \infty$ can be directly derived.

Appendix B: Mathematical Formalization of the Generalized Distance Function

In the following, we provide the mathematical formalization of the generalized distance function (described in detail in Berkowitsch et al., 2013). This generalized distance function is meant to describe the distance between options in the multiattribute space, distinguishes the preferential relationship between options, and accounts for individual differences by incorporating the subjective weights that individuals give to different attributes in the distance function.

We define an importance weight vector $\boldsymbol{W}$, which contains the individual weights of the n attributes and restricts the weights to sum to 1. Further, each indifference vector $\{\boldsymbol{iv}_j\}_{j=1}^{n-1}$ is an n-dimensional vector and can be calculated as

$$\boldsymbol{iv}_j = \begin{bmatrix} -\dfrac{w_{j+1}}{w_1} \\ 0 \\ \vdots \\ 0 \\ \dfrac{w_1}{w_1} \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} -\dfrac{w_{j+1}}{w_1} \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \forall\, j = 1,\ldots,n-1, \qquad \text{(B1)}$$

where 1 is at the $(j + 1)^{\text{th}}$ position.

Because we want the n-dimensional dominance vector $\boldsymbol{dv}$ to be orthogonal to all $n - 1$ indifference vectors, it fulfills

$$\boldsymbol{iv}_j \cdot \boldsymbol{dv} = 0 \forall j = 1,\ldots,n-1 \qquad \text{(B2)}$$

which leads to the generalized form:

$$\boldsymbol{dv} = \begin{bmatrix} \frac{w_1}{w_1} \\ \frac{w_2}{w_1} \\ \vdots \\ \frac{w_j}{w_1} \\ \vdots \\ \frac{w_n}{w_1} \end{bmatrix}. \tag{B3}$$

Now we can build the $n \times n$ basis $\boldsymbol{B}^*$, containing the $n - 1$ indifference vectors $\boldsymbol{iv_1}$ to $\boldsymbol{iv_{n-1}}$ and the dominance vector $\boldsymbol{dv}$

$$\boldsymbol{B}^* = [\boldsymbol{iv}_1,..\boldsymbol{iv}_j...,\boldsymbol{iv}_{n-1},\boldsymbol{dv}]. \tag{B4}$$

To standardize the lengths of the indifference vectors and the dominance vector to 1, each vector is divided by its Euclidean lengths l_{iv} and l_{dv}, where $\{l_{iv_j}\}_{j=1}^{n-1}$

$$l_{iv_j} = \|\boldsymbol{iv}_j\|_2 \; \forall \; j = 1,..., n-1 \tag{B5}$$

and

$$l_{dv} = \|\boldsymbol{dv}\|_2 . \tag{B6}$$

Thus, we obtain new basis $\boldsymbol{B}$**,** which is

$$\boldsymbol{B} = \left[\frac{\boldsymbol{iv_1}}{l_{iv_1}},...,\frac{\boldsymbol{iv_j}}{l_{iv_j}},...,\frac{\boldsymbol{iv_{n-1}}}{l_{iv_{n-1}}},\frac{\boldsymbol{dv}}{l_{dv}} \right] \tag{B7}$$

where the $n \times n$ matrix $\boldsymbol{B}$ contains the standardized indifference vectors and the standardized dominance vector.

Next, we define the standard distance vector $\boldsymbol{dist}_{stand}$ as the trajectory path between two points, expressed in standard unit vectors. To transform $\boldsymbol{dist}_{stand}$ into the new distance vector $\boldsymbol{dist}_{trans}$, which expresses the trajectory path by the previously introduced indifference vectors and by the dominance vector, make a change of basis:

$$\boldsymbol{dist_{trans}} = \boldsymbol{B}^{-1} \cdot \boldsymbol{dist_{stand}}. \tag{B8}$$

The first $n - 1$ entries of $\boldsymbol{dist}_{\boldsymbol{trans}}$ express the distance in units of each $\boldsymbol{iv_j}$, whereas the last entry of $\boldsymbol{dist}_{\boldsymbol{trans}}$ expresses the distance in units of $\boldsymbol{dv}$. Now we need to calculate the Euclidean length $\boldsymbol{D^2}$ of $\boldsymbol{dist}_{\boldsymbol{trans}}$ and multiply the distance in the dominance direction by a parameter $\boldsymbol{wd} > \mathbf{1}$. This assures that the distance in the dominance direction is weighted more strongly than the distance in the indifference directions. This is computed as follows:

$$\boldsymbol{D^2} = \boldsymbol{dist_{trans}}' \cdot \boldsymbol{A} \cdot \boldsymbol{dist_{trans}} \tag{B9}$$

where $\boldsymbol{A}$ is a $n \times n$ diagonal matrix and is constructed in the following way:

$$\boldsymbol{A_{j,j}} = \begin{Bmatrix} 1, & \text{if } j = 1,\ldots,n-1 \\ \boldsymbol{wd}, & \text{if } j = n \end{Bmatrix} \tag{B10}$$

This assures that only the difference in the dominance direction—the last column of $\boldsymbol{dist}_{\boldsymbol{trans}}$ —is weighted by $\boldsymbol{wd}$. By setting $\boldsymbol{A}$ to the identity matrix (i.e., $\boldsymbol{wd} = 1$), one obtains the standard Euclidean norm.

Appendix C: Constraints on Model Parameters

For the logit model we estimated an importance weight for each of the *n* attributes, denoted w_i. Here, attribute weights were allowed to vary between 0 and ∞. For the probit model we estimated *n* – 1 attribute weights and the variance *v* of the normal distributed error component **ε** in the diagonal of the variance–covariance matrix. Attribute weights were restricted to sum to 1 and the *v* was allowed to vary between 0 and 1,000. Finally, for MDFT we estimated the *n* – 1 attribute weights, restricted to sum to 1, the variance *v* of the normal distributed error component **ε**, restricted to values between 0 and 1,000, the sensitivity parameter φ_1, restricted to vary between 0.01 and 1,000, and the decay parameter φ_2, restricted to values between 0 and 1. In Study 1, we fixed the weight parameter *wd*, which weights distances in the dominance direction relative to distances in the indifference direction, to a value of 12, following Hotaling et al. (2010). In Study 2, *wd* was estimated from the data. Here, the value range was restricted to between 1 and 50 (see the Supplemental Materials for the estimated ranges of all parameters). Prior to parameter estimation, we rescaled the range of the attributes to values between 0 and 1 and recoded all attributes such that higher numbers indicate higher values.

To compute the likelihoods of the models we applied the multivariate normal distribution for MDFT and the probit model and the multivariate logistic distribution for the logit model and minimized the respective summed log-likelihoods for each participant. To search the best-fitting parameters we used maximum likelihood methods implemented in the *R* functions “nlminb” (package: stats) and “psoptim” (package: pso).

Supplemental Materials

Ranges of the Estimated Parameters

In the following Tables S1 to S7 we provide the ranges of the estimated parameters for all models in Studies 1 and 2. For the logit model we estimated the n weights w_i for each attribute i, allowed to vary between 0 and ∞. The parameters used to estimate the probit model were the $n - 1$ weights w_i for $n - 1$ attributes, which were restricted to sum to 1, and the variance v of the normal distributed error component $\boldsymbol{\varepsilon}$ in the diagonal of the variance–covariance matrix, allowed to vary between 0 and 1,000. Finally, for the MDFT we estimated the $n - 1$ weights w_i allocated to the $n - 1$ attributes, restricted to sum to 1, the variance v of the normal distributed error component $\boldsymbol{\varepsilon}$, restricted to values between 0 and 1,000, the sensitivity parameter φ_1, restricted to vary between 0.01 and 1,000, and the decay parameter φ_2, restricted to values between 0 and 1. In Study 1, we fixed the weight parameter ***wd***, which weights distances in the dominance direction relative to distances in the indifference direction, to a value of 12, following Hotaling et al. (2010) and we estimated its value in Study 2, restricting values to between 1 and 50.

Table S1

Study 1—Logit Model: Estimated Parameter Ranges of the Attribute Weights (w_1-w_5)

Quantile	w_1	w_2	w_3	w_4	w_5
$Q_{0.25}$	1.31	0.55	1.46	0.24	0.59
$Q_{0.50}$	2.14	1.09	2.86	1.01	0.92
$Q_{0.75}$	3.61	1.70	4.84	3.01	1.64

Table S2

Study 1—Probit Model: Estimated Parameter Ranges of the Attribute Weights (w_1-w_5) and Variance (v)

Quantile	w_1	w_2	w_3	w_4	w_5	v
$Q_{0.25}$	0.15	0.05	0.20	0.01	0.06	0.02
$Q_{0.50}$	0.24	0.13	0.30	0.10	0.09	0.02
$Q_{0.75}$	0.41	0.19	0.45	0.29	0.14	0.03

Table S3

Study 1—MDFT: Estimated Parameter Ranges of the Attribute Weights (w_1-w_5), Variance (v), Decay Parameter (φ_2), and Sensitivity Parameter (φ_1)

Quantile	w_1	w_2	w_3	w_4	w_5	v	φ_2	φ_1
$Q_{0.25}$	0.10	0.03	0.19	0.05	0.07	0.27	0.02	0.14
$Q_{0.50}$	0.19	0.10	0.33	0.09	0.10	0.88	0.09	0.42
$Q_{0.75}$	0.40	0.21	0.50	0.23	0.18	1.64	0.22	1.68

Table S4

Study 2—Logit Model: Estimated Parameter Ranges of the Attribute Weights (w_1-w_2)

Quantile	Digital camera		Color printer		Notebook computer		Racing bike		Vacuum cleaner		Washing machine	
	w_1	w_2	w_1	w_2	w_1	w_2	w_1	w_2	w_1	w_2	w_1	w_2
$Q_{0.25}$	5.67	2.90	7.52	4.98	3.46	8.78	7.35	7.59	8.14	4.35	9.37	0.20
$Q_{0.50}$	10.80	17.52	12.49	11.15	8.41	15.31	14.12	14.48	19.59	13.52	24.00	8.46
$Q_{0.75}$	25.23	30.62	34.57	18.60	18.83	30.81	28.30	34.01	38.67	24.14	62.52	23.53

Table S5

Study 2—Probit Model: Estimated Parameter Ranges of the Attribute Weights (w_1-w_2) and Variance (v)

Quantile	Digital camera			Color printer			Notebook computer			Racing bike			Vacuum cleaner			Washing machine		
	w_1	w_2	v	w_1	w_2	v	w_1	w_2	v	w_1	w_2	v	w_1	w_2	v	w_1	w_2	v
$Q_{0.25}$	0.33	0.04	0.00	0.37	0.20	0.00	0.00	0.58	0.00	0.27	0.24	0.00	0.19	0.11	0.00	0.41	0.05	0.00
$Q_{0.50}$	0.59	0.41	0.01	0.59	0.41	0.01	0.24	0.76	0.02	0.49	0.51	0.00	0.58	0.42	0.01	0.62	0.38	0.01
$Q_{0.75}$	0.96	0.67	0.08	0.80	0.63	0.21	0.42	1.00	0.40	0.76	0.73	0.04	0.89	0.81	0.19	0.95	0.59	0.69

Table S6

Study 2—MDFT: Estimated Parameter Ranges of the Attribute Weights (w_1-w_2), Variance (v), Decay Parameter (φ_2), Sensitivity Parameter (φ_1), and Dominance Parameter wd

Quantile	Digital camera			Color printer			Notebook computer			Racing bike			Vacuum cleaner			Washing machine			φ_2	φ_1	*wd*
	w_1	w_2	v	w_1	w_2	v	w_1	w_2	v	w_1	w_2	v	w_1	w_2	v	w_1	w_2	v			
$Q_{0.25}$	0.30	0.05	0.01	0.46	0.26	0.01	0.12	0.48	0.02	0.33	0.36	0.03	0.39	0.27	0.03	0.53	0.06	0.01	0.05	28.68	1.09
$Q_{0.50}$	0.67	0.33	0.09	0.57	0.43	0.11	0.36	0.64	0.11	0.45	0.55	0.10	0.58	0.42	0.18	0.67	0.33	0.11	0.05	58.68	3.47
$Q_{0.75}$	0.95	0.70	1.16	0.74	0.54	0.47	0.52	0.88	3.30	0.64	0.67	1.84	0.73	0.61	0.51	0.94	0.47	0.56	0.06	93.20	11.85

Table S7

Study 2—MDFT: Estimated Parameters of Selected Participants with RST > .50 and PRST > .50 Simultaneously for Attraction, Compromise, and Similarity Choice Triplets

Participant	Digital camera			Color printer			Notebook computer			Racing bike			Vacuum cleaner			Washing machine			φ_2	φ_1	*wd*
	w_1	w_2	v	w_1	w_2	v	w_1	w_2	v	w_1	w_2	v	w_1	w_2	v	w_1	w_2	v			
3	0.40	0.60	0.01	0.52	0.48	0.11	0.45	0.55	0.28	0.34	0.66	0.00	0.40	0.60	0.08	0.55	0.45	0.10	0.05	120.58	9.19
10	0.00	1.00	0.00	0.94	0.06	0.48	0.44	0.56	0.13	0.11	0.89	0.16	0.52	0.48	0.07	1.00	0.00	0.00	0.05	105.94	1.04
38	1.00	0.00	0.00	1.00	0.00	0.00	0.36	0.64	0.02	0.62	0.38	0.20	0.73	0.27	0.10	0.40	0.60	0.14	0.05	71.01	1.43

Reducing the MDFT to a Probit Model

This section shows how MDFT can be transformed into a probit model. Technically, this can be achieved by setting the feedback matrix and by omitting the variance–covariance matrix of the product-relevant attention weights.

One of the main differences between MDFT and a probit model is the dynamic aspect of MDFT. To reduce MDFT to a static model, such that the preference state **P** remains constant over time, we set the feedback matrix **S** to a zero matrix. A zero feedback matrix indicates no previous memory of previous preference states and no competition between the options (IIA property). Technically, a zero matrix for **S** can be achieved by setting φ_2 to 1 (no memory of the previous preference state) and $\varphi_1 \rightarrow \infty$.

In a next step, we set the variance–covariance matrix of the product-relevant attributes **Ψ** to zero, such that only the variance–covariance matrix of the product-irrelevant attributes remains. **Ψ** is calculated within the variance–covariance matrix of the valence vector **V**. As described in Roe and colleagues (2001), the valence vector **V** consists of the valence produced by product-relevant attributes [$\mathbf{CM_1W_1}(t)$] and the valence of the product-irrelevant attributes ([$\mathbf{CM_2W_2}(t) = \boldsymbol{\varepsilon}(t)$] where **C** is a contrast matrix to compute the momentary advantage or disadvantage of each option relative to the average of the other options. $\mathbf{M_1}$ represents a $J \times n$ subjective evaluation matrix, where J are the number of options and n are the number of attributes describing option j. $\mathbf{W_1}(t)$ is an $n \times 1$ vector containing the attention weights of each attribute i at time point t. The variance–covariance matrix of **V** can be expressed as (Roe et al., 2001)

$$Cov\left[\mathbf{V}(t)\right] = Cov\left[\mathbf{CM}_1\mathbf{W}_1(t) + \boldsymbol{\varepsilon}(t)\right] = \mathbf{CM}_1 Cov\left[\mathbf{W}_1(t)\right]\mathbf{M}_1'\mathbf{C}' + Cov\left[\boldsymbol{\varepsilon}(t)\right]$$

$$= \mathbf{CM}_1\boldsymbol{\Psi}\mathbf{M}_1'\mathbf{C}' + \varsigma = \Phi \tag{S1}$$

where

$$\boldsymbol{\Psi} = Cov\left[\mathbf{W}_1(t)\right] = E\left[(\mathbf{W}_1(t) - \mathbf{w}_1)(\mathbf{W}_1(t) - \mathbf{w}_1)'\right] \tag{S2}$$

is the variance–covariance matrix for the product-relevant weights and

$$\varsigma = Cov\left[\boldsymbol{\varepsilon}(t)\right] = E\left[\boldsymbol{\varepsilon}(t)\boldsymbol{\varepsilon}(t)'\right] = \text{diag}\ (\boldsymbol{\varepsilon}(t)) \tag{S3}$$

is the variance–covariance matrix of the product-irrelevant weights. If we assume that the attention shifts according to a Bernoulli process in an all-or-none manner from one attribute to another (see Appendix B of Roe et al., 2001), the probability of focusing on the product-relevant attributes can be written as

$$\mathbf{w}_1 = E\left[\mathbf{W}_1(t)\right]. \tag{S4}$$

Accordingly, $\boldsymbol{\Psi}$ reduces to

$$\boldsymbol{\Psi} = \text{diag}\ (\mathbf{w}_1) - \mathbf{w}_1\mathbf{w}_1'. \tag{S5}$$

By setting $\boldsymbol{\Psi}$ to a zero matrix, we reduce the variance–covariance matrix of $\mathbf{V}$ to ς. To achieve this, we multiply $\boldsymbol{\Psi}$ by the scalar 0, which otherwise is set to 1. This turns $\mathbf{CM}_1 Cov\left[\mathbf{W}_1(t)\right]\mathbf{M}_1'\mathbf{C}'$ into a zero matrix. Thus, only the covariance of the product-irrelevant attributes ς remains. The assumption that the product-irrelevant attributes are uncorrelated with the product-relevant attributes and uncorrelated with each other implies that $Cov[\boldsymbol{\varepsilon}]$ is a diagonal matrix. Thus the variance–covariance matrix of the valence vector ($\mathbf{V}$) can be rewritten as

$$Cov\left[\mathbf{V}(t)\right] = Cov\left[\mathbf{V}\right] = Cov\left[\mathbf{CM}_1\mathbf{W}_1 + \boldsymbol{\varepsilon}\right] = 0 + Cov\left[\boldsymbol{\varepsilon}\right] = \text{diag}\ (\boldsymbol{\varepsilon})\ , \tag{S6}$$

which is the same as the variance–covariance matrix of the (independent) probit model.

How Previous Choices Affect Present Choices:

Explaining Choice Dependencies with a Sequential Sampling Model

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Abstract

Past choices can influence subsequent choices, for instance, to show consistency or to compensate for mistakes. Many examples of such sequence effects have been reported in the literature, especially in moral decision making research. We also found sequence effects in an online employee selection experiment ($N = 126$). In this personnel selection task the choice of job candidates strongly depended on previously chosen candidates. To explain the cognitive processes underlying sequence effects, we propose a Dependent Sequential Sampling Model (DSSM). To incorporate the influence of previous choices, the DSSM builds on current Sequential Sampling Models (SSMs) and rests on three main assumptions. First, previous choices with little confidence are more likely to result in compensating behavior, whereas previous choices with high confidence are more likely to result in consistent behavior. Second, recent choices have a stronger impact on subsequent choices as compared to more remote choices. Third, whether compensating or consistent choice results from a previous choice varies individually. The empirical results and the suggested model illustrate how dependencies in decision making should be taking into account to predict people's choice behavior.

Keywords: sequential sampling model, preferential choice, sequential decision making, employee selection

Imaging yourself standing in an elevator. You see a person running toward the closing doors. You would have enough time to press the “open” button. Would you? If you did, would you do it again for a second person that comes around the corner soon afterwards and again for a third person? Although you are not in a hurry, you might eventually think that you have been "good enough" and just let the doors close. Rather than making isolated, independent choices, previous choices often influence our subsequent choice. The goal of the present work is to illustrate dependencies in decision making and to explain the underlying cognitive process of these dependencies with a proposed sequential sampling model.

Research on sequence effects provides many examples where previous choices influence subsequent choices. For example, Schwarz (2011) analyzed the referees’ penalty decisions for the German football league over the last 40 years (1963-2006). In matches with two penalty decisions, the team which was awarded with the first penalty was less likely to be awarded with second penalty as well. Such influence from previous choices is also prevalent for gambling decisions. According to the gambler’s fallacy people often think that a winning color will win with lower probability in the next trial (e.g., Ayton & Fischer, 2004; Clotfelter & Cook, 1993). In consequence, people often switch their bet to the other color. Or people, who imagined to have volunteered for a charitable act or to have donated money to a charity organization, were subsequently more likely to choose the more hedonic consumer good (e.g., designer jeans, luxury sunglasses; Khan & Dhar, 2006). Besides the prevalence in consumer choices, such sequence effects have been reported in many domains, such as, risky choice behavior (e.g., Thaler & Johnson, 1990; Novemsky & Dhar, 2005), categorization research (e.g., Stewart & Brown, 2004), food choices (Effron, Monin, & Miller, 2013), and moral decision making (e.g., Monin & Miller, 2001).

Current cognitive models, such as reinforcement learning models, provide an approach to explain sequence effects (e.g. Kruschke, 1992, Gluck & Bower, 1988; Rieskamp & Otto, 2006; Simão & Todd, 2002; Stewart, Brown, Chater, 2002; Stewart & Brown, 2004; Todd, 2007). They assume that based on feedback on task performance people learn and adapt their choice behavior. For example, according to the strategy selection learning theory feedback on the task performance reinforces correctly chosen strategies (Rieskamp & Otto, 2006). Similarly, Stewart et al. (2002) propose with their memory and contrast model that–among other factors–feedback on previous choices strongly influence subsequent choices. In their experiment they asked participant to indicate to which of two categories an acoustic tone belongs. They found that feedback on the correct classification of previous tones, strongly influenced their subsequent classification. However, sequence effects can also occur without learning and in situations without any feedback. For instance, substantial research in moral decision making illustrated sequence effects even in the absence of explicit feedback. In the following we will illustrate these effects.

Sequence Effects: Previous Choices Influence Subsequent Choices

Moral decision making research has shown that after a morally dubious choice people feel the urge to subsequently engage in a virtuous choice, which has been labeled as *cleansing* behavior (Tetlock, Kristel, Elson, & Lerner, 2000). Thereby, the act of cleansing serves as a mean to maintain a moral self-image and to reaffirm moral values (Sachdeva, Illiev, & Medin, 2009; Tetlock et al., 2000). For example, participants who were asked to write down an unethical deed compared to an ethical deed, chose more frequently antiseptic wipe over a pencil (Zhong & Liljenquist, 2006, Study 3). Further, after describing an unethical deed, people who did not cleanse their hands with an antiseptic compared to those who did, were more likely to subsequently offer their help to a graduate student (Zhong & Liljenquist, 2006, Study 4). Zhong, Ku, Lount, and Murnighan (2010) showed that merely asking people to imagine a dubious choice, leads people to subsequently engage in more virtuous choices. Conversely, a virtuous or a moral choice can *license* a subsequent morally dubious choice (Monin & Miller, 2001). The virtuous choice affirms people's moral identity and in consequence they feel licensed to subsequently act in a selfish way (Sachdeva et al., 2009).

For example, when people were given the opportunity to represent themselves as free from racial prejudice, they were more likely to hire a White man than a Black man in a subsequent choice task (Monin & Miller, 2001). Likewise, Effron, Miller, and Monin (2012) showed in an experiment that if participants could point to past immoral roads not taken, it "licensed" them to act in morally dubious ways later on. The authors argue that if people can establish “moral credentials” as being unprejudiced, they fear less being attributed as sexists or racists (Monin & Miller, 2001). For examples of licensing behavior in consumer choices see Khan and Dhar (2006) and for food choices see Effron, Monin, and Miller (2012). Both, licensing and cleansing behavior can be characterized as *compensating* choice behavior (Conway & Peetz, 2012; Jordan, Mullen, & Murnighan, 2011; Sachdeva et al., 2009): Either to compensate for a previous bad deed (i.e., cleansing) or for a previous good deed (i.e., licensing).

In contrast, previous virtuous deeds can also motivate *consistent* choice behavior. For example, Gneezy, Imas, Nelson, Brown and Norton (2012, Experiment 2) reported that visitors of an amusement park who purchased a souvenir photo in the charity condition, were subsequently more likely to buy a present for others (i.e., a prosocial behavior) than for themselves (i.e., a selfish behavior). The authors proposed that people infer their own morality based on the costliness of their past behavior leading to consistent subsequent choices in the case of previous costly behavior. In a different study Zhang, Cornwell, and Higgins (2013) found prevention-focused individuals–individuals who seek to maintain the status quo–favor consistent choice behavior and were more likely to cheat in a second task after having cheated in a first task.

These empirical findings on compensating and consistent choice behavior nicely illustrate the prevalence of sequence effects outside of performance tasks, yet they have two limitations. First, to elicit the sequence effects, researchers used different tasks within their experiments. For example, in the first task of the experiment by Monin and Miller (2001) participants were asked to rate negative statements about women, before they had to indicate in a second task whether a job opening was better suited for one gender. Although it is important to know that sequence effect can occur across tasks, many of our daily duties involve repeated choices in a similar environment. A judge often pronounces multiples sentences a day, or a human resource manager has to choose repeatedly between multiple job candidates. Thus, it is important to examine whether sequence effects (i.e., compensating choice behavior) also occur in repeated choice tasks. Second, the proposed theories to predict and explaining compensating choice behavior have an exclusively verbal status that do not allow for quantitative predictions (for a verbal model on compensatory ethics see Zhong et a., 2010). For testing competing theories against each other formulized computational models are advantageous (e.g., Lewandowsky & Farrell, 2011).

The remaining of the paper is organized as follow. The next section addresses the first limitation, where we test whether compensating choice behavior occurs in a repeated choice task (i.e., job selection task). The findings will provide a basis to the second challenge, namely to provide a cognitive approach to model potential compensating and consistent choice behavior. This will be the focus of the subsequent section where we formalize a cognitive model following sequential sampling models. In the final section we discuss further applications and limitations of the model.

Sequential Choice Experiment

We conducted an experiment in which participants repeatedly choose between two job candidates, varying in gender. Previous research on job selection in a moral context found that only participants who could disagree with sexist or racist statements (e.g., “Men are more emotionally suited for politics than are most women”), subsequently indicated a (White) male to be better qualified for a job opening (e.g., Monin and Miller, 2001). In a recent study Effron et al. (2012) reported similar findings. Participants had to indicate which of two suspects was more likely to have committed a crime. If participants were given a "racist" alternative (i.e., having a very unlikely Black suspect and a White suspect), they were more likely to subsequently indicate that a White person was more qualified for a job opening, than if this alternative was not present (i.e., having only White suspects). Whereas in these previous experiments the choice task followed a rating task, participants in our experiment were faced with repeated choice tasks.

In hiring situations there is a standard social norm that women compared to men are discriminated and are hired less often, especially when being equally qualified as men. Therefore, repeatedly hiring a female candidate against the standard prejudice view will affirm one's moral identity, and as a consequence people will feel licensed to make a subsequent choice that seems prejudiced (i.e., hiring a male candidate). Thus, if people were to apply such a "licensing strategy", we should observe compensating choice behavior after they repeatedly hired a female candidate. Alternatively, people could subsequently hire a male candidate after repeatedly choosing female candidate as means of balancing the number of female and male candidates. In contrast to the licensing strategy, this "balancing strategy" predicts compensating choice behavior not only after repeatedly choosing female candidates, but also after repeatedly choosing male candidates (Dhar, Huber, & Khan, 2007; Dhar & Simonson, 1999; Huber, Goldsmith, Mogilner, 2008). In line with the balancing strategy is the suggested compensatory ethics model by Zhong et al. (2010). According to their model, people compensate a previous virtuous deed with a subsequent selfish deed (i.e., licensing behavior) and a previous selfish deed with a subsequent virtuous deed (i.e., cleansing behavior), following in the long-run an equilibrium approach.

Thus, if we observe sequence effects, it is immanent for cognitive models to account for the influence of previous choices on subsequent choices, even if the choice situation does not provide explicit feedback. Note the main goal of this experiment is to test whether sequence effects are observed in a repeated choice task in the first place, rather than to exhaustively identify the different strategies people apply.

Method

Participants.

We recruited participants living in the U.S. through amazon's Mechanical Turk (Amazon, 2013). To avoid inattentive participants or computer programs filling out the questions, we included an additional test page at the beginning that instructed participants to ignore the subsequent questions and to just type “I read the instructions” into a text box. Of the 147 (77%) participants who passed that screening, 126 (35% females) finished the experiment. On average the experiment took 6 minutes and participants received 0.30 USD for completing the task.

Procedure and experimental design.

Participants were presented with a hypothetical recruiting scenario and asked to repeatedly choose the most suitable candidate out of two job applicants. All candidates were described by the three attributes "leadership skills", "social competence", and "typing speed" (see Appendix for the exact values), of which the first two were described as important attributes, whereas the third was described as a less important attribute. The profile picture indicated the gender of the candidates. To make sure participants paid attention to the profile picture, they were instructed to also check candidates for a well-groomed appearance (which all of them had).

Each participant made ten pairwise decisions in a row. The description of the task emphasized that the ten candidate pairs applied for ten different companies, thus stressing that each decision ought to be independent of the previous ones. The choice pairs were constructed such that one of the two candidates was clearly superior on the two most important attributes (i.e. leadership skills and social competence) in the first eight pairs (= dominating pairs), whereas the candidates in the last two pairs (i.e., the 9th and the 10th pair) were approximately equally well suited for the job (= non-dominating pairs; see Appendix for the exact values). The non-dominating pairs always consisted of a male and a female candidate, whereas the gender of the candidates within the dominating pairs varied between the four between-subjects conditions (see Table 1) outlined next.

In the first *female>male* condition the dominating candidate was always female and the dominated candidate was always male. In the second *male>female* condition the dominating candidate was always male and the dominated candidate was always female. In the third *female>female* condition all candidates were female and in the fourth *male>male* condition all candidates were male. The only condition in which participants can affirm their moral identity and establish moral credentials is in the *female>male* condition, as it is the only condition in which participants are given the opportunity to hire a (dominating) female candidate over a (dominated) male candidate. Given that participants have previously chosen the dominating candidate, a licensing strategy predicts the *choice share of the male candidate (CSM)* in the subsequent non-dominating pairs to be highest in the *female>male* condition as compared to the other three conditions. On the other hand, if participants apply a balancing strategy, we should observe a higher *CSM* in the *female>male* condition and in the *female>female* condition as compared to the *male>female* condition and the *male>male* condition.

The participants were randomly assigned to one of the four conditions. The attribute values describing the ten candidate pairs were the same for all conditions. To rule out order effects, we randomized the orders within the eight dominating pairs and the two non-dominating pairs across participants.

Results and Discussion: Choice Shares of the Candidate Pairs

As shown in Figure 1, most participants across all four experimental conditions chose the dominating candidates in the first eight pairs, indicating that participants did pay attention to the provided information. To detect possible sequence effects, we first compared whether the *CSM* in the *female>male* condition was higher than the *CSM* in each condition for the non-dominating pairs. We tested these differences against 0, applying a Bayesian analysis.

Thereto, we contrasted their 95% highest density interval (HDI) representing the most credible posterior *CSM* values. For all conditions we used uniform priors (for details on the Bayesian model see Kruschke, 2011, Ch. 8.5). If the HDI$_{95}$ is positive and excludes 0, we can infer that the *CSM* in the *female>male* condition is higher than the *CSM* in the other conditions.

In line with our prediction, 70% of participants in the *female>male* condition chose to hire the male candidate for the 9th choice pair, as compared to only 40% in the *male>female* condition (HDI$_{95}$ 0.06 – 0.49; 99% of the HDI > 0), 33% in the *female>female* condition (HDI$_{95}$ 0.12 – 0.56; 100% of the HDI > 0), and 25%, in the *male>male* condition (HDI$_{95}$ 0.20 – 0.63; 100% of the HDI > 0). The *CRM* in the *female>female* condition was not higher than in the *male>female* condition (HDI$_{95}$ -0.29 – 0.16; 29% of the HDI > 0) than in the *male>male* condition (HDI$_{95}$ -0.15 – 0.30; 75% of the HDI > 0). This is in line with previous research, which shows that only participants, who could establish moral credentials, were more likely to subsequently choose the male candidate (Monin & Miller, 2001).

To test for possible sequence effects within the two non-dominating pairs, we compared the *CSM* of the 9th against the 10th choice pair across conditions. The previously observed choice pattern reversed for most conditions. In the *female>male* condition the *CSM* dropped to 33% (HDI$_{95}$ 0.12 – 0.56; 100% of the HDI < 0), whereas it increased in the *female>female* condition to 60% (HDI$_{95}$ -0.48 – -0.02; 98% of the HDI < 0), and to 64% in the *male>male* condition (HDI$_{95}$ HDI$_{95}$ -0.59 – -0.13; 100% of the HDI < 0). Only for the *male>female* condition the *CSM* of the 10th choice pair was not different from the 9th choice pair (43%; HDI$_{95}$ -0.24 – 0.20; 59% of the HDI < 0). These reversed choice patterns suggest that participants followed a licensing strategy for the first non-dominating pair and followed a balancing strategy for the second non-dominating pair (see also Zhong et al., 2010).

From these results we can infer three important points: First, sequence effects occur in repeated choice tasks and not only across different tasks, indicating that people take previous choices into account when facing a new similar decision. This is even the case when the consecutive choices should be independent. Remember, that we told our participants to select candidates for different companies, thus, stressing independent choices. Second, the observed compensating choice behavior is rather attributed to a licensing strategy than to a balancing strategy. This suggests that simply choosing a female candidate does not necessarily elicit compensating choice behavior. Only when the dominating female candidate was paired with a dominated male candidate, but not when paired with a female candidate, we observed compensating choice behavior. Third, whereas people prefer a licensing strategy in the short run, they appear to apply a balancing strategy in the long run.

Together with previous findings, our results emphasize the importance to incorporate the history of our previous choices into choice models. However, so far, current theories that aim to explain these effects rely on verbal descriptions; more precise cognitive models that build on mathematical grounds are lacking.

A cognitive Approach to model Sequence Effects

Towards a cognitive model that accounts for the influence of previous choices, we propose the Dependent Sequential Sampling model (DSSM). The DSSM builds upon a broader class of sequential sampling models (SSMs), that aim to explain the underlying cognitive process of decision making (Busemeyer & Townsend, 1993; Ratcliff, 1978; Townsend & Ashby, 1983; Usher & McClelland, 2004; Vickers, 1970), outlined next.

Sequential Sampling Models

SSMs have become particularly popular in describing the cognitive process underlying decision making. They have been applied to explain decision process underlying perceptual, inferential, preferential and risky choices, to name a few (Berkowitsch, Scheibehenne, & Rieskamp, in press; Trueblood, 2012; Trueblood, Brown, Heathcote, & Busemeyer, 2013; Tsetsos, Chater, and Usher, 2012). SSMs assume that people accumulate evidence of preference for each available option (for suggested accumulation processes see Brown &

Heathcote, 2008; Ratcliff, 1978; Roe, Busemeyer, & Townsend, 2001; Usher & McClelland, 2004). The preference of the option first to pass a specific threshold is chosen. These models further allow accounting for previous (good or bad) experiences with the options by allowing individual initial preferences for the options which is set to a constant value (i.e., usually to 0 if a decision maker had no previous experience with the option). Because the model we propose in this paper strongly builds on the basic framework of SSMs, we start by introducing Multialternative Decision Field Theory (MDFT; Roe et al., 2001), a prominent SSM in more detail.

MDFT assumes that preferences for each option under consideration are accumulated over time, whereas the preference state (denoted $\boldsymbol{P}_t$) at any given point in time integrates all previous preferences states of that option. According to the model the attention given to attributes describing the options (e.g., leadership skills, socials skills) switches in an all-or-nothing manner (Roe et al., 2001). People evaluate the momentary valences of the options by comparing the options' focal attribute values (e.g., leadership skills performance) relative to each other and weight by the subjective importance $\mathbf{w}$ they assign to the attributes. These valences are then added to the options' previous preference state underlying decay forming $\boldsymbol{P}_t$ (for the exact formula of this updating process see Roe et al.).

Once the accumulated preference of one option passes a specific threshold $\boldsymbol{\theta}$, this option (e.g., the female candidate) is chosen (internal stopping-rule). Alternatively, one might assume that decision makers accumulate evidence for a fixed amount of time and then choose the option that is most preferred at the end of the time frame (external stopping-rule). Similarly, it is also possible to assume that evidence is accumulated until the options' preferences have converged to a stable state (converging stopping-rule; see Berkowitsch et al., 2013).

The preferences difference $\Delta\boldsymbol{P}$ at the time of the decision can also be used as an indicator of the confidence in the decision (Lee & Dry, 2006; Vickers, 1979; Vickers & Lee, 1998), where smaller differences imply lower level of confidence. The initial preference $\boldsymbol{z}$ for each option reflects previous experience with the options. Figure 2 illustrates a possible accumulation process of MDFT when each options is equally likely to be chosen at the beginning (i.e., $\boldsymbol{z} = 0$).

The other parameters of MDFT influence the preference accumulation process. These are, the variance v of the normal distributed error, the rate parameter φ_1 at which similarity declines with distance between the options, the decay parameter φ_2 and the dominance parameter *wd*, reinforcing distance in dominance direction. Because we do not focus on the accumulation process, these parameters will not be discussed any further (for details see Berkowitsch et al., 2013; Roe et al., 2001; Hotaling, Busemeyer, & Li, 2010).

SSMs, such as the MDFT, provide no explicit theory that specify the influence of previous choices on present choices, but often assume that choices are made independent from each other. Whereas the assumption of independence might be reasonable for many choice situations, the examples above indicate that there are also many occasions in which previous choices exert a systematic influence on subsequent choices.

In the following we explain how the SSMs can be extended to account for sequence effects. We first present a verbal description of the DSSM, followed by its mathematical formalization. Subsequently, we apply DSSM on our experimental data and compare it to MDFT which assumes independent choices. Finally, we discuss further applications of DSSM.

DSSM: Incorporating previous Choices

Previous choices can *inform* subsequent choices, thereby changing people's experience with the available options. From the perspective of SSMs, this influence could be modeled as a change in the initial preference $\boldsymbol{z}$ for subsequent choices. Thus, instead of setting $\boldsymbol{z}$ constant, we define $\boldsymbol{z}$ as a function of previous choices. To account for compensating and consistent choice behavior we propose three cognitive mechanisms:

First, we propose to adjust the initial preference state such that previous choices made with little confidence are more likely to result in compensating choice behavior, whereas previous choices that were made with high confidence are more likely to result in consistent choice behavior. Second, because human memory underlies decay recent as compared to more remote choices should have a stronger effect on present choices. A decay parameter accounts for individual differences in the decay process. Third, whereas for some individuals a previous choice might result in compensating choice behavior, for others the same choice might yield consistent or even reinforcing choice behavior. To account for these individual differences, we included a compensating-consistency parameter in the decay process.

Thus, the change $\Delta\boldsymbol{z}$ of the initial preference state $\boldsymbol{z}$ is determined by the previous levels of confidence, a decay function, and an individual compensating-consistency parameter. These three processes, and how DSSM incorporates them, are described in the following in more detail.

Mathematical Formalization of DSSM

The level of confidence $\Delta\boldsymbol{P}_i$ after the i^{th} choice can be expressed as the difference between the preferences states of two options *F* (e.g., choosing the female candidate) and *M* (e.g., choosing the male candidate), whereas smaller differences imply lower level of confidence. Following MDFT (Roe et al. 2001), we assume that the preference states $\boldsymbol{P}_t$ associated with the two options sum to 0. We will omit the subscript *t* for convenience and refer to $\boldsymbol{P}$ as the final preference state prior to a decision. The *i*-dimensional vector $\Delta\boldsymbol{\xi}_i$ contains the previous levels of confidence up to the i^{th} choice:

$$\Delta\boldsymbol{\xi}_i = \begin{bmatrix} \Delta\boldsymbol{P}_1 \\ \Delta\boldsymbol{P}_2 \\ \dots \\ \Delta\boldsymbol{P}_i \end{bmatrix} = \begin{bmatrix} \boldsymbol{P}_{1,F} - \boldsymbol{P}_{1,M} \\ \boldsymbol{P}_{2,F} - \boldsymbol{P}_{2,M} \\ \dots \\ \boldsymbol{P}_{i,F} - \boldsymbol{P}_{i,M} \end{bmatrix}, \forall\, i = 1, \dots, N, \tag{1}$$

where *N* is the total number of choices. Coded that way, choosing option *F* results in the i^{th} choice in a positive $\Delta\boldsymbol{P}_i$, whereas choosing option M results in a negative $\Delta\boldsymbol{P}_i$.

The decay process is captured by an *i*-dimensional decay vector $\boldsymbol{\rho}_i$, indicating to what extent each previous $\Delta\boldsymbol{P}_i$ is taken into account when faced with a new decision. In line with past research on memory processes, we assume exponential decay (e.g., Brown, Neath, & Chater, 2007; Page & Norris, 1998; Shepard, 1987):

$$\boldsymbol{\rho}_i = \boldsymbol{\kappa} \cdot e^{-\boldsymbol{\lambda} \cdot \boldsymbol{decay}_i}, \forall\, i = 1, \ldots, N \qquad (2)$$

The compensation-consistency parameter $\boldsymbol{\kappa}$ $(-\infty < \boldsymbol{\kappa} < \infty)$ moderates the influence of the previous choices on the subsequent choice. Setting $\boldsymbol{\kappa} > 0$ promotes *compensating* choice behavior, whereas setting $\boldsymbol{\kappa} = 0$ promotes *consistent* choice behavior. Setting $\boldsymbol{\kappa} < 0$ reflects *reinforcing* choice behavior. A prominent example for reinforcing choice behavior in the literature is the foot-in-the-door-effect (Burger, 1999; Freedman & Frazer, 1966) according to which previous commitments to helping requests increases the likelihood of subsequent commitments.

The decay parameter $\boldsymbol{\lambda}$ $(0 < \boldsymbol{\lambda} < \infty)$ determines how quickly the decay vector $\boldsymbol{\rho}_i$ decreases from $\boldsymbol{\kappa}$ to 0. Higher values of $\boldsymbol{\lambda}$ indicate stronger decay such that only the very last choices are taken into account. The decreasing decay values are captured by the *i*-dimensional vector $\boldsymbol{decay_i}$:

$$\boldsymbol{decay_i} = -\begin{bmatrix} 1 \\ 2 \\ \vdots \\ i \end{bmatrix} + \begin{bmatrix} i \\ i \\ \vdots \\ i \end{bmatrix} = \begin{bmatrix} -1+i \\ -2+i \\ \vdots \\ -i+i \end{bmatrix} = \begin{bmatrix} -1+i \\ -2+i \\ \vdots \\ 0 \end{bmatrix}; \forall\, i = 1, \ldots, N. \qquad (3)$$

Along with the exponential function, $\boldsymbol{decay_i}$ assures exponentially decreasing decay values toward 0, such that the very recent choice has the strongest influence ($e^{-\lambda \cdot (0)} = 1$) and more remote choices have a weaker influence (i.e., $e^{-\lambda \cdot (0)} > \cdots > e^{-\lambda \cdot (-2+i)} > e^{-\lambda \cdot (-1+i)}$) on subsequent choices. The same process holds true for $\boldsymbol{\kappa} < 0$, except that the decay values increase toward 0.

Figure 3 illustrates this process of the decay vector $\boldsymbol{\rho}_i$ for hypothetical individuals after having made for sequential choices. The individual $\boldsymbol{\rho}_i$ contains the different levels of influence the previous choices exert on the subsequent 5th choice as a function of $\boldsymbol{\lambda}$ and $\boldsymbol{\kappa}$. For example, $\boldsymbol{\kappa} = 1$ and $\boldsymbol{\lambda} = 0.5$ indicates a case in which all four previous choices influence the subsequent choice, whereas for $\boldsymbol{\kappa} = 1$ and $\boldsymbol{\lambda} = 2$ the first two choices only have a marginal influence on the subsequent choice (see Figure 3A). Setting $\boldsymbol{\kappa} = 0$ indicates a case for which independent choices are assumed (Figure 3C).

Next, we combine the three cognitive mechanisms into one function to determine the change of the initial preference state $\Delta \boldsymbol{z}_i$ between the ith and the i + 1th choice. Because we assume that the influence of the previous levels of confidence is moderated by the decay process, $\Delta \boldsymbol{z}_i$ is:

$$\Delta \boldsymbol{z}_i = \begin{cases} -\left[\frac{1}{\Delta \xi_i}\right]^{\mathrm{T}} \cdot \boldsymbol{\rho}_i & if\ \Delta \boldsymbol{P}_i = \boldsymbol{P}_{i,F} - \boldsymbol{P}_{i,M} \\ +\left[\frac{1}{\Delta \xi_i}\right]^{\mathrm{T}} \cdot \boldsymbol{\rho}_i & if\ \Delta \boldsymbol{P}_i = \boldsymbol{P}_{i,M} - \boldsymbol{P}_{i,F} \end{cases} \tag{4}$$

$\forall\ i = 1, \ldots, N$,

where N is the total number of choices and the component-wise division of 1 by $\Delta \boldsymbol{\xi}_i$ assures that with decreasing level of confidence, compensating behavior becomes more likely. From Equation (4) follows that for $\boldsymbol{\kappa} > 0$ and $\Delta \boldsymbol{P}_i = \boldsymbol{P}_{i,F} - \boldsymbol{P}_{i,M}$, choosing option F over option M in the ith choice results in $\Delta \boldsymbol{z}_i < 0$

Finally, to account for the influence of previous choices on the subsequent choice between options F and M, DSSM assumes that the initial preferences for the two options are updated according to the following rule:

$$\boldsymbol{z}_{i+1} = \begin{bmatrix} \boldsymbol{z}_{i+1,F} \\ \boldsymbol{z}_{i+1,M} \end{bmatrix} = \begin{bmatrix} \boldsymbol{z}_{i,F} + \Delta \boldsymbol{z}_i \\ \boldsymbol{z}_{i,M} - \Delta \boldsymbol{z}_i \end{bmatrix} = \boldsymbol{z}_i + \Delta \boldsymbol{z}_i \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} \tag{5}$$

$\forall\ i = 1, \ldots, N$; $\boldsymbol{z}_1 = 0$,

where N is the total number of choices.

From Equations (4) and (5) follow that for $\boldsymbol{\kappa} > 0$ and $\Delta \boldsymbol{P}_i = \boldsymbol{P}_{i,F} - \boldsymbol{P}_{i,M}$, choosing option F over option M in the ith choice results in $\Delta \boldsymbol{z}_i < 0$ which leads to $\boldsymbol{z}_{i+1,F} < \boldsymbol{z}_{i+1,F}$ and $\boldsymbol{z}_{i+,M} >$

$\boldsymbol{z}_{i+1,M}$. In other words choosing option *F* decreases the subsequent initial preference of option *F* and increases the subsequent initial preference of option *M*, if $\boldsymbol{\kappa}$ is set to positive values (i.e., compensating choice behavior). Conversely, if instead an individual choses option *M* over option *F*, $\boldsymbol{z}_{i+1,F} > \boldsymbol{z}_{i+1,F}$ and $\boldsymbol{z}_{i+1,M} < \boldsymbol{z}_{i+1,M}$, thus increasing the subsequent initial preference of option *F* and decrease the subsequent initial preference of option *M*. When setting $\boldsymbol{\kappa} = 0$, then $\boldsymbol{z}_i = \boldsymbol{z}_{i+1}$ because $\Delta\boldsymbol{z}_i = 0$, indicating consistent or independent choice behavior, as it is the case for MDFT. Thus, MDFT is nested within DSSM. Finally when $\boldsymbol{\kappa}$ is set to negative values and $\Delta\boldsymbol{P}_i = \boldsymbol{P}_{i,F} - \boldsymbol{P}_{i,M}$, choosing option *F* increases the likelihood of choosing option *F* again and decreases the likelihood of subsequently choosing option *M*.

Applying DSSM to the Experimental Data

Can DSSM account for the observed sequence effects observed in the experimental data described above? The *female>male* condition indicated that, for most participants a preference reversal toward the male candidate occurred for the 9th choice pair. We simulated this choice pattern based on the actual attribute values of the ten job candidate pairs, using the DSSM and compared it to MDFT. For the data on hand, we applied an external stopping-rule, but other stopping-rules are applicable too.

In line with the instructions in the experiment, we assumed equal weights for leadership skills and social competence ($w_{LS} = w_{SC} = .5$) and assigned typing speed a weight of zero ($w_{TS} = 0$). First, to simulate the decision makers who first chose the male candidate for the 9th choice pair and subsequently the female candidate in the 10th choice pair, we set $\boldsymbol{\kappa}$ = .04 and $\boldsymbol{\lambda}$ = 1. The resulting choice patterns for DSSM and MDFT are shown in Figure 4. The difference between the two models becomes particularly clear for the 9th and 10th choice. Whereas MDFT assigns both job candidates the same preference states (Figure 4C), DSSM favors the male candidate in the 9th and the female candidate in the 10th candidate (Figure 4A). If we instead assume an even stronger compensating behavior, we can increase the value of $\boldsymbol{\kappa}$, for example to $\boldsymbol{\kappa}$ = 0.2. Holding the other parameters constant DSSM now predicts that the male candidate is preferred in both the 9th and 10th choice (Figure 4B). The lower $\boldsymbol{\kappa}$ value in Figure 4A compared to 4B yields a lower level of confidence after the 9th choice, making a subsequent preference reversal in the 10th choice more likely. Notice that both choice patterns shown in Figures 4A and 4B cannot be predicted by MDFT (Figure 4C).

General Discussion

Rather than making independent choices, past research indicated that previous choices often influence subsequent choices. Such dependencies have been found in many domains including risky options (Camerer, 1989; Gilovich, Vallone, & Tversky, 1985), choices among foods (Dhar & Simonson, 1999), or moral actions (Monin & Miller, 2001; Tetlock et al., 2000). Here, we further showed that sequence effects can also occur in an employee selection tasks. In particular, we showed that compensating choice behavior can also occur for repeated choice tasks. So far, current cognitive models of choice, including SSMs, cannot account for these sequential dependencies, as these choice situations neither provide any explicit feedback on the accuracy of the decision nor learning opportunities.

The present work shows that DSSM can account for compensating, consistent, and reinforcing choice behavior in a choice environment not providing any feedback. Instead of setting the initial preference of an option constant over choices, DSSM assumes that initial preferences are updated based on our previous choices. The proposed updating function incorporates three cognitive mechanisms. First, the model assumes that lower level of confidence makes preference reversals more likely. On the other hand higher level of confidence motivates consistent choices behavior. Second, DSSM assumes a decay process and takes the number of previous choices, as well as their individual impacts on the subsequent choice into account. Third, whether compensating, consistent, or even reinforcing choice behavior result from previous choice is subject to the choice situation and the decision maker. These individual differences are captured by a compensating-consistency parameter in the decay process.

Besides DSSM's ability to incorporate sequence effects, its capability to account for individual differences provides the advantage to predict a variety of empirical effects observed in the choice literature. For example, research on social influence suggests that people differ in their need for consistency (for a review see Guadagno & Cialdini, 2010). Since the mathematical sign of $\boldsymbol{\kappa}$ either reflects compensating or consistent choice behavior, future research could correlate $\boldsymbol{\kappa}$ with people's measure of preference for consistency (Cialdini, Trost, & Newsome, 1995). Other scholars have identified the abstractness of sequential choices and the time span between sequential choices as possible moderators of compensating and consistent choice behavior (Conway & Peetz, 2012; Huber et al., 2008).

DSSM can also be applied to predict reinforcing choice behavior observed in the literature. For example, according to the foot-in-the-door-effect, the likelihood to agree on a second bigger request increases after agreeing to smaller first request (Freedman & Frazer, 1966). On the other hand repeated unethical decision making can become more likely after a first unethical decision. Zhang et al. (2013) found that people who are motivated to maintain a status quo (i.e., prevention focus) are more likely to cheat in a subsequent task after cheating in the initial task. Both of these choices behaviors could be captured by DSSM by setting $\boldsymbol{\kappa} < 0$, increasing the likelihood of an individual to repeat a previous action (e.g., to help someone or to cheat) by increasing its subsequent initial preference. Again, it would be interesting to correlate $\boldsymbol{\kappa}$ with the psychological processes associated with the foot-in-the-door-effect or with measures of individuals' prevention focus (Burger, 1999; Higgins, 1997).

Finally, DSSM could be applied to account for the gambler's fallacy, according to which people ascribe a winning number a lower probability of a subsequent win (Clotfelter & Cook, 1993) and its counterpart, the hot hand fallacy (e.g. Scheibehenne, Wilke, & Todd, 2011). Although the consecutive spins of a roulette wheel are independent from each other, there is ample evidence showing that gamblers' bets are often influenced by previous observations. Instead of assuming independence and thus assigning each outcome the same initial preference, the initial preferences could be updated contingent on past outcomes according to the above described mechanisms.

One limitation of our work is that we did not estimate the parameters of the model from the empirical data, but used simulations to show how DSSM can account for compensating and consistent choice behavior. To account for these sequence effects we introduced new parameters which raises the question whether increasing the model complexity is justified by a higher model fit. Thus, future work should test DSSM against other SSMs, such as MDFT. As the number of observed compensating choice behaviors increases the DSSM eventually outperforms the other SSMs. Future work could also extend DSSM to multiple choices and include a no-choice option.

To define DSSM we made several assumptions on the model which future work should test empirically. First, we assumed that lower choice confidence increases a subsequent preference reversal (i.e., compensating choice behavior), whereas higher choice confidence increases consistent choice behavior. Future research could test these predictions by eliciting people's choice confidence. Second, we assumed that the influence of previous choices follows an exponential decay. Other scholars propose that decay processes follow rather a power-law (Anderson & Schooler, 1991; Donkin & Nosofsky, 2012; Wixted & Ebbesen, 1991). An empirical model comparison could test which of two versions of DSSM provides a better model fit, one version following an exponential decay, the other using a power-law. Third, DSSM assumes that $\boldsymbol{z}$ is updated whenever $\boldsymbol{\kappa} \neq 0$. From this follows a testable prediction that if an individual applies an internal stopping-rule, the deliberation time t should decrease, as the required accumulated preference of an option to pass a specific threshold $\boldsymbol{\theta}$ decreases. This prediction could be tested by assessing participants' response times.

To conclude, our empirical findings support the previous findings on sequence effects, that people are influenced by previous choices when faced with a new decision. Neglecting this cognitive process can result in inaccurate predictions of people's choice behavior. This emphasizes the importance to incorporate previous choices into cognitive choice models. We offered a solution to this proposing DSSM. To get back to the elevator example, people familiar with DSSM might no longer be irritated of closing elevator doors, as they will infer that the people inside the elevator must have previously pressed the "open" button rather often.

References

Amazon (2013). Retrieved from https://requester.mturk.com/

Anderson, J. R., & Schooler, L. J. (1991). Reflections of the environment in memory. *Psychological Science, 2*, 396-408.

Aquino, K., & Reed, A., 2nd. (2002). The self-importance of moral identity. *Journal of Personality and Social Psychology*, *83*, 1423–1440.

Ayton, P., & Fischer, I. (2004). The hot hand fallacy and the gambler's fallacy: Two faces of subjective randomness? *Memory and Cognition*, *32*, 1369–1378. doi:10.3758/BF03206327

Berkowitsch, N. A. J., Scheibehenne, B., & Rieskamp, J. (in press). Rigorously testing Multialternative Decision Field Theory against standard random utility models. *Journal of Experimental Psychology: General*.

Brown, G. D. A., Neath, I., & Chater, N. (2007). A temporal ratio model of memory. *Psychological Review*, *114*, 539–576. doi:10.1037/0033-295X.114.3.539

Brown, S. D., & Heathcote, A. (2008). The simplest complete model of choice response time: Linear ballistic accumulation. *Cognitive Psychology*, *57*, 153–178. doi:10.1016/j.cogpsych.2007.12.002

Burger, J. M. (1999). The foot-in-the-door compliance procedure: A multiple-process analysis and review. *Personality and Social Psychology Review*, *3*, 303–325. doi:10.1207/s15327957pspr0304_2

Busemeyer, J. R., & Townsend, J. T. (1993). Decision field theory: A dynamic-cognitive approach to decision making in an uncertain environment. *Psychological Review*, *100*, 432–459.

Camerer, C. F. (1989). Does the basketball market believe in the `Hot Hand,'? *The American Economic Review*, *79*, 1257–1261. doi:10.2307/1831452

Cialdini, R. B., Trost, M. R., & Newsom, J. T. (1995). Preference for consistency: The development of a valid measure and the discovery of surprising behavioral implications. *Journal of Personality and Social Psychology*, *69*, 318–328. doi:10.1037/0022-3514.69.2.318

Clotfelter, C. T., & Cook, P. J. (1993). Notes: The "Gambler's Fallacy" in lottery play. *Management Science*, *39*, 1521–1525. doi:10.1287/mnsc.39.12.1521

Conway, P., & Peetz, J. (2012). When does feeling moral actually make you a better person? Conceptual abstraction moderates whether past moral deeds motivate consistency or compensatory behavior. *Personality and Social Psychology Bulletin*, *38*, 907–919. doi:10.1177/0146167212442394

Dhar, R., Huber, J., & Khan, U. (2007). The shopping momentum effect. *Journal of Marketing Research, 44*, 370-378.

Dhar, R., & Simonson, I. (1999). Making complementary choices in consumption episodes: Highlighting versus balancing. *Journal of Marketing Research*, *36*, 29–44. doi:10.2307/3151913

Donkin, C. & Nosofsky, R. M. (2012). A power-law model of psychological memory strength in short- and long-term recognition. Psychological Science, 23, 625-634. doi: 10.1177/0956797611430961

Effron, D. A., Miller, D. T., & Monin, B. (2012). Inventing racist roads not taken: The licensing effect of immoral counterfactual behaviors. *Journal of Personality and Social Psychology*, *103*, 916–932. doi:10.1037/a0030008

Effron, D.A., Monin, B, & Miller, D. T. (2013). The unhealthy road not taken: Licensing indulgence by exaggerating counterfactual sins. *Journal of Experimental Social Psychology, 49*, 573-587.

Freedman, J. L., & Fraser, S. C. (1966). Compliance without pressure: The foot-in-the-door technique. *Journal of Personality and Social Psychology*, *4*, 195–202. doi:10.1037/h0023552

Gilovich, T., Vallone, R., & Tversky, A. (1985). The hot hand in basketball: On the misperception of random sequences. *Cognitive Psychology*, *17*, 295–314. doi:10.1016/0010-0285(85)90010-6

Gluck, M. A., & Bower, G. H. (1988). From conditioning to category learning: an adaptive network model. *Journal of Experimental Psychology General*, *117*, 227–247.

Gneezy, A., Imas, A., Brown, A., Nelson, L. D., & Norton, M. I. (2012). Paying to be nice: Consistency and costly prosocial behavior. *Management Science, 58*, 179-187.

Guadagno, R. E., & Cialdini, R. B. (2010). Preference for consistency and social influence: A review of current research findings. *Social Influence*, *5*, 152–163. doi:10.1080/15534510903332378

Higgins, E. T. (1997). Beyond pleasure and pain. *American Psychologist, 52*, 1280-1300.

Hotaling, J. M., Busemeyer, J. R., & Li, J. (2010). Theoretical developments in decision field theory: Comment on Tsetsos, Usher, and Chater (2010). *Psychological Review*, *117*, 1294–1298. doi:10.1037/a0020401

Huber, J., Goldsmith, K., & Mogilner, C. (2008). Reinforcement versus balance response in sequential choice. *Marketing Letters, 19*, 229-239. doi: 10.1007/s11002-008-9042-5

Jordan, J., Mullen, E., & Murnighan, J. K. Striving for the moral self. The effects of recalling past moral actions on future moral behavior. *Personality and Social Psychology Bulletin, 37*, 701-713. doi: 10.1177/0146167211400208

Khan, U., & Dhar, R. (2006). Licensing effect in consumer choice. *Journal of Marketing Research*, *43*, 259–266. doi:10.2307/30163392

Kruschke, J. K. (1992). ALCOVE: An exemplar-based connectionist model of category learning. *Psychological Review*, *99*, 22–44.

Kruschke, J. K. (2011). Bayesian assessment of null values via parameter estimation and model comparison. *Perspectives on Psychological Science, 6*, 299-312. doi: 10.1177/1745691611406925

Lee, M. D., & Dry, M. J. (2006). Decision making and confidence given uncertain advice. *Cognitive Science, 30*, 1081-1095.

Monin, B., & Miller, D. T. (2001). Moral credentials and the expression of prejudice. *Journal of Personality and Social Psychology*, *81*, 33–43. doi:10.1037/0022-3514.81.1.33

Novemsky, N., & Dhar, R. (2005). Goal fulfillment and goal targets in sequential choice. *Journal of Consumer Research*, *32*, 396–404. doi:10.1086/497551

Page, M. P. A., & Norris, D. (1998). The primacy model: A new model of immediate serial recall. *Psychological Review*, *105*, 761–781. doi:10.1037/0033-295X.105.4.761-781

Ratcliff, R. (1978). A theory of memory retrieval. *Psychological Review*, *85*, 59–108. doi:10.1037/0033-295X.85.2.59

Reed, A., 2nd, & Aquino, K. F. (2003). Moral identity and the expanding circle of moral regard toward out-groups. *Journal of Personality and Social Psychology*, *84*, 1270–1286.

Rieskamp, J., & Otto, P. E. (2006). SSL: A theory of how people learn to select strategies. *Journal of Experimental Psychology: General*, *135*(2), 207–236. doi:10.1037/0096-3445.135.2.207

Roe, R. M., Busemeyer, J. R., & Townsend, J. T. (2001). Multialternative decision field theory: A dynamic connectionist model of decision making. *Psychological Review*, *108*, 370–392.

Sachdeva, S., Iliev, R., & Medin, D. L. (2009). Sinning saints and saintly sinners: The paradox of moral self-regulation. *Psychological Science*, *20*, 523–528. doi:10.1111/j.1467-9280.2009.02326.x

Scheibehenne, B., Wilke, A., & Todd, P. M. (2011). Expectations of clumpy resources influence predictions of sequential events. *Evolution and Human Behavior, 32*, 326-333. doi:10.1016/j.evolhumbehav.2010.11.003

Shepard, R. N. (1987). Toward a universal law of generalization for psychological science.

Science, 237, 1317-1323.

Simao, J., & Todd, P. M. (2002). Modeling mate choice in monogamous mating systems with courtship. *Adaptive Behavior*, *10*, 113–136. doi:10.1177/1059712302010002003

Stewart, N., & Brown, G. D. A. (2004). Sequence effects in the categorization of tones varying in frequency. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *30*, 416–430. doi:10.1037/0278-7393.30.2.416

Stewart, N., Brown, G. D. A., & Chater, N. (2002). Sequence effects in categorization of simple perceptual stimuli. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *28*, 3–11. doi:10.1037/0278-7393.28.1.3

Tetlock, P. E., Kristel, O. V., Elson, S. B., Green, M. C., & Lerner, J. S. (2000). The psychology of the unthinkable: Taboo trade-offs, forbidden base rates, and heretical counterfactuals. *Journal of Personality and Social Psychology*, *78*, 853–870. doi:10.1037//0022-3514.78.5.853

Thaler, R. H., & Johnson, E. J. (1990). Gambling with the house money and trying to break even: The effects of prior outcomes on risky choice. *Management Science*, *36*, 643–660. doi:10.1287/mnsc.36.6.643

Todd, P. M. (2007). How much information do we need? *European Journal of Operational Research*, *177*, 1317–1332. doi:10.1016/j.ejor.2005.04.005

Townsend, J. T., & Ashby, F. G. (1983). *The stochastic modeling of elementary psychological processes*. CUP Archive.

Trueblood, J. S. (2012). Multialternative context effects obtained using an inference task. *Psychonomic Bulletin & Review, 19*, 9962-9968. doi: 10.3758/s13423-012-0288-9

Trueblood, J. S., Brown, S. D., Heathcote, A., & Busemeyer, J. R. (2013). Not just for consumers: Context effects are fundamental to decision making. *Psychological Science, 24*, 901-908. doi: 10.1177/0956797612464241

Tsetsos, K., Chater, N., & Usher, M. (2012). Salience driven value integration explains decision biases and preference reversals. *Proceedings of the National Academy of Sciences, 109,* 9659-9664. doi: 10.1073/pnas.1119569109

Usher, M., & McClelland, J. L. (2004). Loss aversion and inhibition in dynamical models of multialternative choice. *Psychological Review*, *111*, 757–769. doi:10.1037/0033-295X.111.3.757

Vickers, D. (1970). Evidence for an accumulator model of psychophysical discrimination. *Ergonomics*, *13*, 37–58. doi:10.1080/00140137008931117

Vickers, D. (1979). *Decision processes in visual perception.* New York, NY: Academic Press.

Vickers, D., & Lee, M. D. (1998). Dynamic models of simple judgments: I. Properties of a self-regulating accumulator module. *Nonlinear Dynamics, Psychology, and Life Sciences, 2*, 169-194.

Wixted, J. T., & Ebbesen, E. B. (1991). On the form of forgetting. *Psychological Science, 2*, 409-415.

Zhang, S., Cornwell, J. F. M., & Higgins, E. T. (2013). Repeating the past: Prevention focus motivate repetition, even for unethical decision. *Psychological Science*. Advance online publication. doi: 10.1177/0956797613502363

Zhong, C.-B., Ku, G., Lount, R. B., & Murnighan, J. K. (2010). Compensatory ethics. *Journal of Business Ethics*, *92*, 323–339. doi:10.1007/s10551-009-0161-6

Zhong, C.-B., & Liljenquist, K. (2006). Washing away your sins: Threatened morality and physical cleansing. *Science*, *313*, 1451–1452. doi:10.1126/science.1130726

Table 1

Overview of the four experimental conditions.

Condition	Dominating pairs				Non-dominating pairs	
	1th pair	2th pair	…	8th pair	9th pair	10th pair
female>male	♀ > ♂	♀ > ♂	♀ > ♂	♀ > ♂	♀ ≈ ♂	♀ ≈ ♂
male>female	♂ > ♀	♂ > ♀	♂ > ♀	♂ > ♀	♀ ≈ ♂	♀ ≈ ♂
female>female	♀ > ♀	♀ > ♀	♀ > ♀	♀ > ♀	♀ ≈ ♂	♀ ≈ ♂
male>male	♂ > ♂	♂ > ♂	♂ > ♂	♂ > ♂	♀ ≈ ♂	♀ ≈ ♂

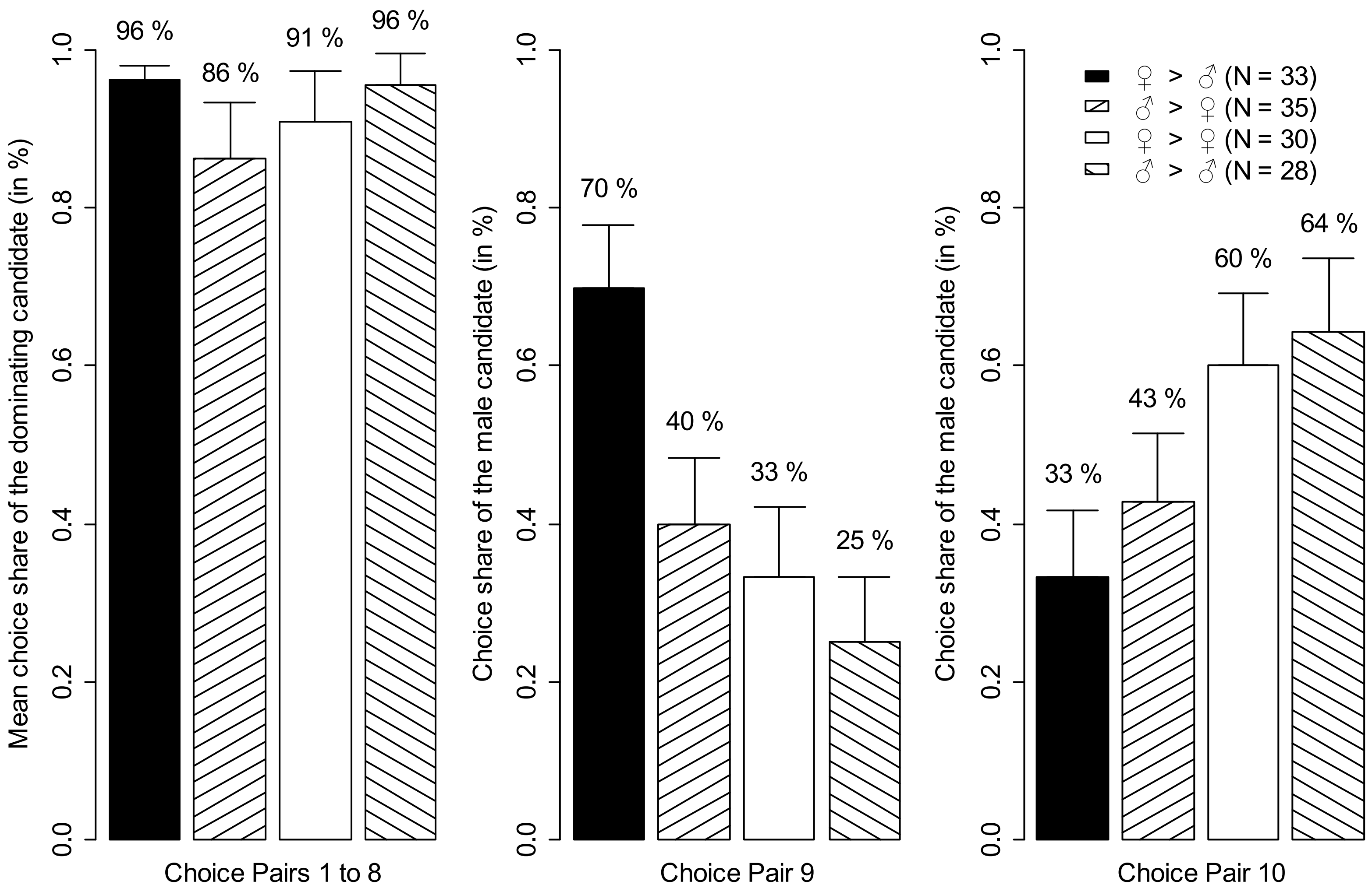

Figure 1. Mean choice shares of the dominating candidate (pairs 1-8) and the male candidate (pairs 9-10) across conditions. Error bars indicate the standard error of the mean.

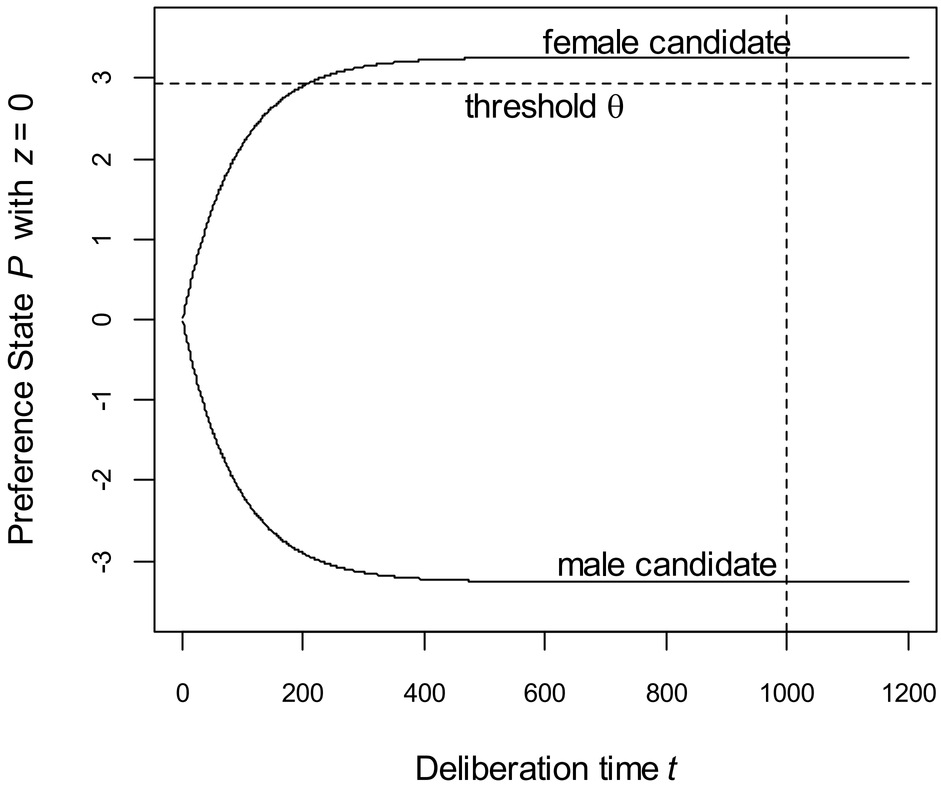

Figure 2. Example of the preference accumulation process of MDFT, according to which an individual chooses the female over the male candidate. The dotted lines indicate different stopping-rules: internal stopping-rule (horizontally dotted line), external stopping rule (vertical dotted line).

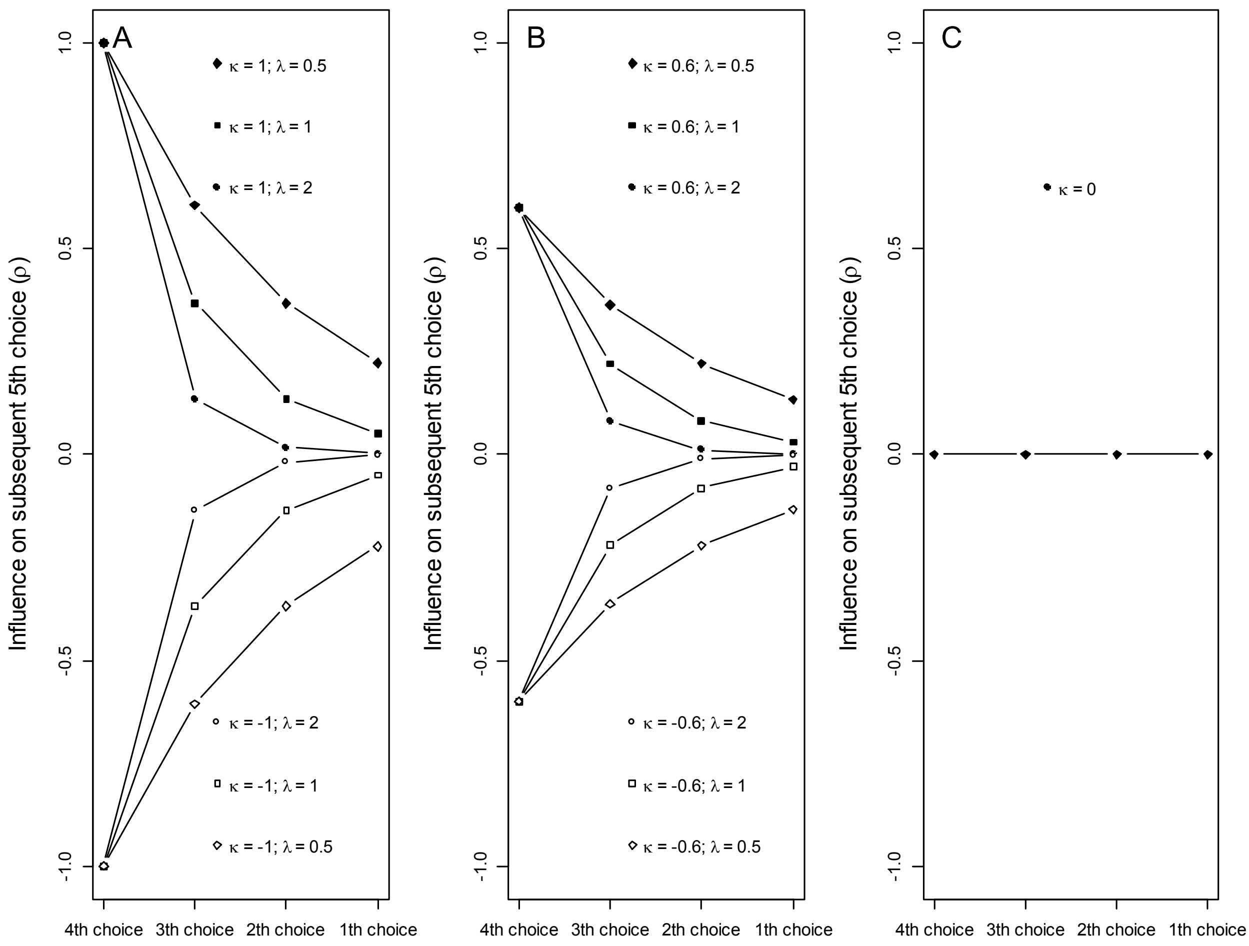

Figure 3. The decay vector $\boldsymbol{\rho}_i$ as a function of different parameter values of $\boldsymbol{\lambda}$ and $\boldsymbol{\kappa}$ after four sequential choices. The different lines indicate individual $\boldsymbol{\rho}_i$ of hypothetical individuals after having made four sequential choices. In both Figures 3A and B more recent choices (i.e., 4^{th} choice) are assumed to exert a stronger influence on the subsequent 5^{th} choice as compared to more remote choices (i.e., 1^{th} choice), but with stronger influence in Figure 3A than Figure 3B. Figure 3C provides an example in which sequential choices are assumed to be independent from each other ($\boldsymbol{\kappa}$ = 0).

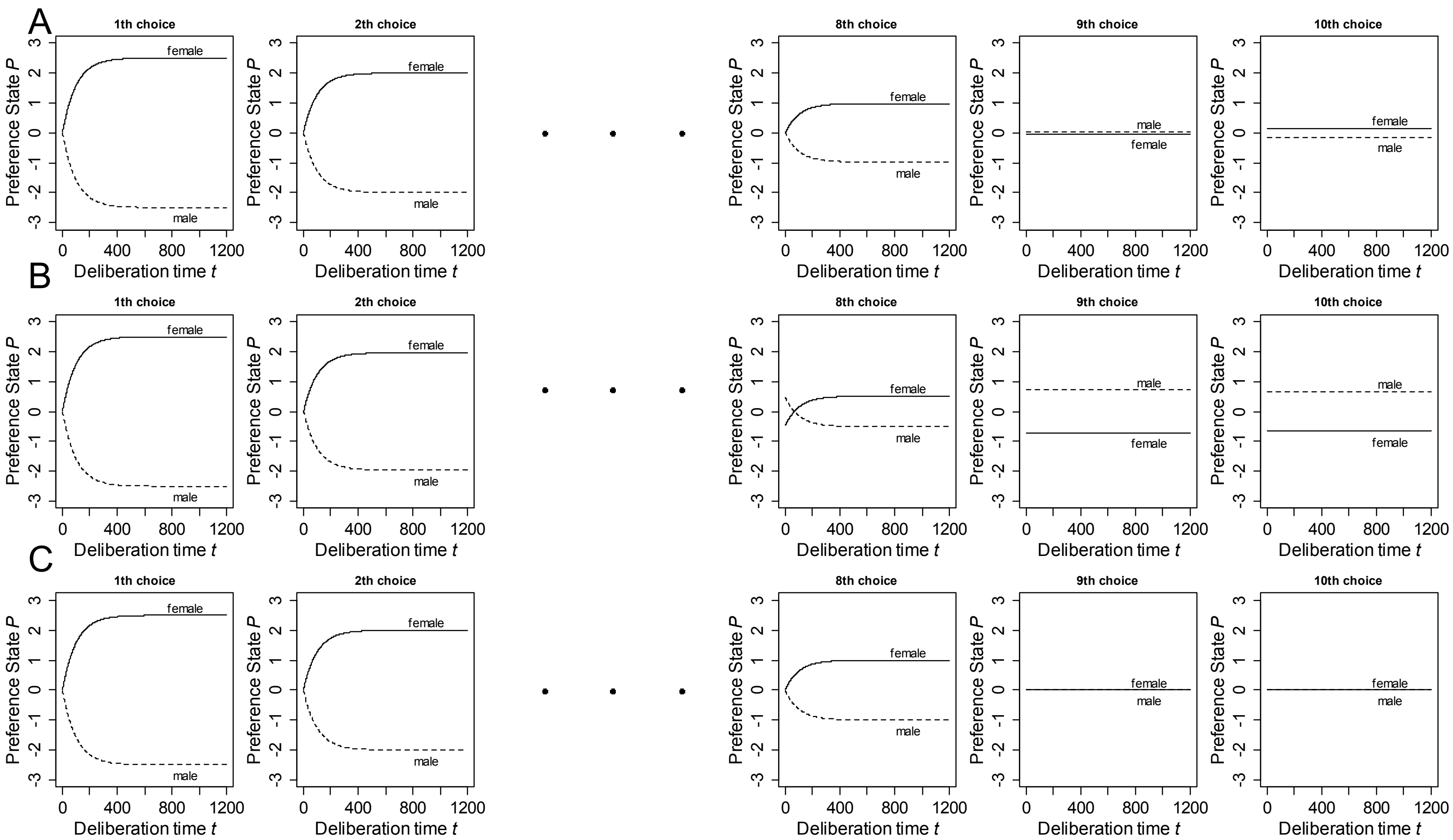

Figure 4. Simulated preference evolution based on the values of the *female>male* condition with a preference reversal after the 8th (Figures 4A and B) choice for DSSM, but not MDFT (Figure 4C) (Used parameter values for the models: $w_{LS} = .5$, $w_{SC} = .5$, $w_{TS} = 0$, $v = 1$; $\varphi_2 = .05$; $\varphi_1 = .46$, $\kappa_A = .04$, $\kappa_B = 0.2$, $\kappa_C = 0$, $\lambda = 1$, $wd = 12$).

Appendix: Attribute Values used for the Dominating and Non-Dominating Job Pairs

Dominating Pairs	Candidate	Leadership Skills	Social Competence	Typing Speed
1	A	41	65	72
	B	45	66	70
2	A	79	75	41
	B	82	76	42
3	**A**	72	27	56
	B	68	27	57
4	A	50	50	44
	B	54	51	48
5	**A**	80	74	38
	B	77	73	39
6	A	39	35	41
	B	43	36	43
7	A	28	64	73
	B	31	66	72
8	**A**	48	71	80
	B	47	70	78
Non-Dominating Pairs				
1	A	52	41	39
	B	50	43	41
2	A	76	70	68
	B	74	72	70

Note. The orders within the dominating and non-dominating pairs were randomized. The choice pair number indicates its position (1 to 8 for the dominating pairs; 9 to 10 for the non-dominating pairs). The letter in boldface indicates the dominating candid