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Substitution between Natural Gas and Electricity

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Jan Abrell, Hannes Weigt

Corresponding Author:

Prof Dr. Hannes Weigt
Forschungsstelle Nachhaltige Energie- und Wasserversorgung
Wirtschaftswissenschaftliche Fakultät der Universität Basel
Peter Merian-Weg 6
Postfach, CH-4002 Basel
Tel. +41 (0)61 267 3259
Fax +41 (0)61 267 0496
hannes.weigt@unibas.ch

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Investments in a Combined Energy Network Model: Substitution between Natural Gas and Electricity?

Jan Abrell[*] **Hannes Weigt**[†]

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Natural gas plays an important role in the future development of electricity markets as it is the least emission intensive fossil generation option while additionally providing the needed flexibility in plant operation to deal with intermittent renewable generation. As both the electricity and the natural gas market rely on networks, congestion on one market may lead to changes on another. In addition, investments in one market have an impact in the other and may even become substitutes for one another. The objective of this paper is to develop a dynamic model representation of coupled natural gas and electricity network markets to test the potential interaction with respect to investments. The model is tested under simplified conditions as well as for a stylized European network setting. The results indicate that there is a potential for investment-substitution and significant market interactions that warrants the application of coupled models especially with regard to simulations of long term system developments.

Keywords: Electricity network, Natural gas network, Europe, MCP

1. Introduction

Throughout the world a transition of existing energy systems is supposed to take place in the coming decades. Industrialized countries aim for a switch from fossil based to more renewable energy fueled systems and in developing regions of the world a significant increase of energy demand will occur. Both developments will require a large amount of investments in energy production and transport infrastructure (IEA (2011a) estimates investment needs of ca. 38 trillion $ till 2035). As energy markets are interlinked

[*]Swiss Federal Institute of Technology Zürich, Center for Energy Policy and Economics at ETH Zurich, Zürichbergstrasse 18, CH-8032 Zürich, Switzerland. jabrell@ethz.ch

[†]Corresponding author. Forschungsstelle Nachaltige Energie- und Wasserversorgung, Wirtschaftswissenschaftliche Fakultät der Universität Basel, Peter Merian-Weg 6, Ch-4002 Basel, Switzerland. hannes.weigt@unibas.ch, Phone: +41-(0)61 267 3259, Fax: +41-(0)61 267 0496

with each other due to the substituting character for specific utilization (e.g. heating oil/gas vs. heating with biomass vs. electricity) and the direct usage of energy fuels in downstream markets (e.g. coal, oil and gas as fuel input for electricity) the relations between markets need to be accounted in estimates of future developments. In addition energy markets often rely on network structures which add a spatial layer to the problem.

This interaction of energy markets is particular relevant for natural gas and electricity systems. The increasing importance of emission reductions raises the need for a shift from coal based to natural gas fired units. Similar the increased utilization of intermittent renewable generation units raises the need for more flexible generation units as back-up capacities which are mainly assumed to be gas fired. These developments will likely increase demand for natural gas in the electricity sector which can raise the need for investment in gas infrastructure. On the supply side the development of unconventional gas and further increases in the LNG infrastructure can lead to shifts in the global natural gas market prices (IEA, 2011b) which in turn will have an impact on the dispatch order of existing electricity units and influence investment incentives. As this interaction is becoming increasingly relevant for gas and electricity markets due to the importance of gas as future fuel and the network characteristics of both markets research addressing this interaction has enlarged in recent years. Rubio et al. (2008)provide a review on integrated natural gas and electricity system planning and highlight economic and market related aspects.

A considerable research focus is on the incorporation of natural gas constraints in the short run electricity market dispatch via different modeling approaches. Shahidehpour et al. (2005) accomplish this by focusing this interaction from the electricity market perspective by highlighting the impacts of natural gas infrastructure contingencies on the short run dispatch and price outcomes without a direct model coupling. Several settings are based on two stage models like Urbina and Li (2008) that use a two stage modeling approach with a unit commitment and a network analysis to provide a dispatch and security analyzes with a stylized test system. Similar Li et al. (2008) decompose the setting in a unit commitment model as mixed integer problem and a network analysis as sub-problem. Liu et al. (2009) also use the unit commitment as master problem but provide two sub-problems; the power and natural gas transmission feasibility checks. Mello and Ohishi (2006) develop a dispatch model of gas and electricity using an iterative approach coupling a classical dispatch model with a nonlinear natural gas flow model.

Another approach is the hub system applied by Geidl and Andersson (2005), Koeppel and Andersson (2009) and Krause et al. (2011). It allows incorporating different energy carriers while accounting for flows within the hub (e.g. gas to electricity conversion) and between the hubs (i.e. networking restrictions). Other approaches include a hybrid approach with evolutionary strategy algorithm (e.g. Unsihuay et al., 2007), formulations of integrated markets using the network simplex method (e.g. Gil et al., 2003; Quelhas et al., 2006), including the dynamic aspects of natural gas velocity via quasi dynamic linear modeling (Damavandi et al., 2011), and deriving a single model representation (e.g An et al., 2003, applying primal-dual interior-point methods).

Contrary to the optimization approaches above, Abrell and Weigt (2012) provide a single model representation based on a partial equilibrium market model which includes the interactions of natural gas and electricity networks. The model is time static and applied for a European test case to highlight the interacting effects of down and upstream impacts. The objective of the paper at hand is to extend this time static setting to a dynamic investment representation.

So far only limited approaches exists that address coupled investment settings outside of more generalized energy system model frameworks (e.g. MARKAL or POLES). Unsihuay-Vila et al. (2010) develop a coupled MIP optimal investment model for natural gas and electricity networks but do not account for loop flow characteristics. They apply the model to both a test case and the Brazilian network showing the importance of gas storage in an electricity market with uncertain hydro availability. Lienert and Lochner (2012) combine an electricity model (DIME) and a natural gas model (TIGER). Transmission is represented in a stylized approach based on net-transfer-capacities. They apply the model to the European markets highlighting the short run impact of volatile gas prices and providing long term assessments on gas fired generation capacities. Bakken et al. (2007) develop a coupled model design for multiple energy infrastructures called eTransport. However, the investments are externally defined and ranked by the model approach but no endogenous optimal investment is obtained. Geidl and Andersson (2006) use the hub based approach described above for a structural optimization regarding the conversion technologies in the hubs (i.e. plant technologies) but not the network connecting the hubs.

Chaudry et al (2014) develop a combined network extension model for natural gas and electricity with detailed network flow representations. The model is formulated as cost minimization. Although, a DC-load flow approach is used for the electricity network it is not indicated whether the feedback effect of investments on the flow pattern is accounted in the model. The application to the UK system relies on a linear setup of the British electricity transmission system and therefore does not capture the full nature of meshed network extension.

The objective of this paper is to develop a representation of coupled natural gas and electricity markets focusing on investment options while accounting for the network characteristics of both markets. The underlying model is an extension of the static market representation in Abrell and Weigt (2012) by including time dimensions and investment. The natural gas market is largely characterized by seasonal patterns whereas the electricity market is defined by daily load levels which requires a matching of the two time frames. Investments in natural gas transport infrastructure and power plants follows classical investment representations. However, investments in electricity transmission require a closed formulation of the resulting changes in the networks power transfer distribution factors to capture the impact of meshed networks and loop flows. The dynamic model will be tested under simplified conditions to highlight the impact of meshed electricity networks and the basic substitution effect between gas and electricity. Afterwards a stylized representation of the continental European markets will be analyzed to present potential impacts on future investment developments.

The remainder of this paper is structured as follows. Section 2 provides a general assessment of the investment problem in coupled natural gas and electricity markets. In Section 3 the modeling framework is presented. Following, in Section 4, we first present a numerical illustration of the effects. Afterwards, the European test setting is presented and the scenario results are evaluated. Section 5 summarizes and concludes.

2. The Interaction of natural gas and electricity markets

There are basically two interaction aspects of natural gas and electricity markets. The first is the direct competition as potential fuel for end usage in the heat sector. This is largely driven by the cross price elasticity of both fuels and investment cycles in heating infrastructure particularly for households. This interaction is often represented in CGE frameworks and will not be the scope of this paper.

The second interaction is the linkage of gas as potential fuel for electricity generation and subsequent the bidirectional interdependency on market operation and investment decisions. In separated models this usually becomes sort of a chicken-or-egg problem as assumptions about one market lead to a specific pattern in the other market which in turn can alter the assumed setting in the first place. While this is true for basically all subsequent markets, the natural gas/electricity setting is furthermore complicated by the existence of a network structure making a coupling of both markets within a single model framework more complex and introducing spatial and physical elements.

This interaction and the network dependence can be highlighted when assuming the behavior of an electricity generator:

$$\max_{X \geq 0, CAP \geq 0} \pi = (p - c)\, X - c^{inv} CAP \tag{1}$$

$$\text{s.t} CAP \geq X \quad (\lambda) \tag{2}$$

The generator maximizes its profits (equation 1) by deciding about its generation output (X) and installed generation capacity (CAP) given the market price for electricity (p), its variable generation cost (c) and investment costs (c^{inv}). The installed capacity level restricts the possible generation output (equation 2) and the associated shadow prices λ is given in parenthesis. Besides the capacity restriction, the first order conditions become:

$$c + \lambda \geq p \qquad \perp \quad X \geq 0 \tag{3}$$

$$c^{inv} \geq \lambda \qquad \perp \quad CAP \geq 0 \tag{4}$$

Assuming an interior solution, i.e. generation to be strictly positive, these conditions hold with strict equality: Consequently: $c^{inv} = p - c$. Thus the investment volume in the electricity sector depends on the spread between market prices and generation costs. The market price depends on the location of the plant and therefore includes the network restrictions and the generation costs depend on the plant efficiency η and the fuel price and therefore includes all market aspects on the relevant fuel market. While both aspects can be approximated by using external parameters or simplified market representations this becomes increasingly complicated in large systems and dynamic settings. Furthermore, similar dependencies can be derived for any market participant in the electricity and fuel markets. As electricity markets typically depend on more than one fuel as input due to the different technical characteristics and running hours, and the long lifetimes of installed capacities keeps mixed power plant fleets online for several decades, changes in the underlying fuel prices can easily change the actual market operation.

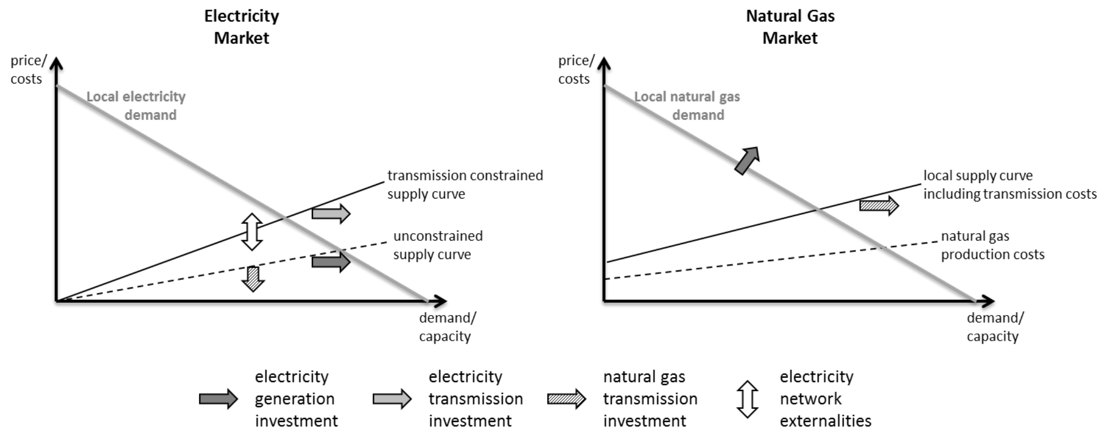

Figure 1: Investment trade-offs in an natural gas based electricity network market

Another perspective to look at the problem is considering the different possibilities and trade-offs when deciding about investments. Within this paper we will focus on two major trade-offs: First, the substitution between electricity network and generation investments and second, the substitution between using natural gas or electricity as transport option.

Figure 1 provides a stylized representation of those different trade-offs. Given a demand function at any location in the electricity market the respective local supply curve is a combination of the unconstrained supply merit-order and the impact of transmission constraints (Figure 1, left panel).[1] The unconstrained supply represents the available generation park in the system. Given network topology and line capacities the local supply curve can deviate from the system wide supply curve due to congestion. As electricity transmission is basically cost free the difference is solely based on limitations of which plants are able to supply a given network node.

Investment in generation capacity will lead to a shift of the unconstrained supply curve towards the right. However, this does not necessarily translate into a price reduction at all network locations. Congestion can reduce the impact of plant capacity additions at specific locations. Similar, an investment in transmission capacity will lead to a shift of the locational transmission constrained supply curve towards the right. Again, relieving congestion does not necessarily lead to price reductions at all network locations.

[1] Given the nature of power generation the supply curve in an electricity market is typically a step-wise, upward sloping function. The presented linear functions are for presentation purpose only. Nevertheless, the same general logic and conclusions also hold for step-wise functions.

Which of the two options is the least cost solution therefore depends on the location in the system and the relevant network and plant restrictions. In addition to this basic trade-off between production and transport investments electricity networks are subject to external effects. As power-flows follow physical laws a change in either the supply conditions of the system (the injection pattern) or the transmission system itself (the lines resistances) will lead to changes in the flow pattern. This in turn can lead to positive as well as negative externalities leading to corresponding shifts in the local supply curve.

The trade-off between natural gas and electricity investments is based on electricity generation. Investments in natural gas transmission will lead to a corresponding shift of the local supply curve in the natural gas market (Figure 1, right panel). This price decrease in natural gas as fuel input for power plants leads to a corresponding downward shift of the electricity supply curve (Figure 1, left panel). Similar, an extension of plant capacities can lead to a shift of the locational natural gas demand function and corresponding price impacts.[2]

Naturally, the gas-electricity trade-off impacts the trade-off between power plant and transmission investments in electricity markets. Within any given electricity-gas market system whether plant extension, electricity or natural gas transmission extensions, or a mixture of those is the optimal solution depends on the specific location within the network(s) and the overall market conditions. Consequently, this setting calls for a combined model approach to capture all those interacting effects.

3. Numerical Framework

The dynamic model setting is formulated as Mixed Complementarity Problems (MCP) and is based on Abrell and Weigt (2012). We provide the optimization setting for each market participants in the natural gas (section 3.1) and electricity (section 3.2) market as well as the market clearing conditions equalizing demand and supply (section 3.3).[3] We assume perfect competition, i.e. all market participants take prices as given. However, the equilibrium concept allows an easy adjustment of the underlying competition assumptions. The MCP model is formulated in the General Algebraic Modeling System (Brooke et al., 2008) and solved using the PATH solver (Ferris and Munson, 2000).

Following, we first describe the sub-model for the natural gas market followed by the model for the electricity market. Afterwards the models are linked using the market clearing equation for natural gas.

3.1. Natural Gas Market

In the natural gas market we explicitly model four market participants: producers, traders, the pipeline operator and final consumers. Natural gas producers extract the gas and sell it to the trader. Only the trader serves final demand by buying natural

[2] Other interactions (i.e. investments in electricity transmission can also lead to changes on the natural gas market demand) are omitted in Figure 1.

[3] The MCP version of the model is provided in Appendix A and the notation is listed in Annex C.

gas and the pipeline transport services necessary to transport it to the final consumers. Consequently, three markets in the natural gas market are explicitly modeled: the supply market, the pipeline transport service market, and the final demand market.

The gas network is described by nodes $g \in \mathcal{G}$ and pipelines given as directed and ordered pairs $(g, \tilde{g}) \in \mathcal{G} \times \mathcal{G}P$ with capacity $cap^{pipe}_{g\tilde{g}}$ [MWh]. Time period in the natural gas model are denoted by $t \in \mathcal{T}^{gas}$. We assume that natural gas demand in period t at node g, DEM^{gas}_{gt} is a linear function of the demand price PD^{gas}_{gt} at that node:

$$DEM^{gas}_{gt} = a^{gas}_{gt} + b^{gas}_{gt} PD^{gas}_{gt} \quad \forall g \in \mathcal{G}, t \in \mathcal{T}^{gas} \tag{5}$$

The natural gas producer at node g maximizes its profit by selling the amount of gas extracted (X^{gas}_{gt}) under the given capacity cap^{gas}_{gt}. It receives the node dependent natural gas supply price PS^{gas}_{gt} and produce under constant marginal cost c^{gas}_{gt}:

$$\max \sum_t \left(PS^{gas}_{gt} - c^{gas}_{gt}\right) X^{gas}_{gt} \tag{6}$$

$$\text{s.t.} \quad cap^{gas}_{gt} \geq X^{gas}_{gt} \quad (PC^{gas}_{gt}) \tag{7}$$

$$X^{gas}_{gt} \geq 0$$

The shadow price on the capacity constraint PC^{gas}_{gt} is provided in parenthesis.

The pipeline trader buys gas at the node of origin $\hat{g}$ at the supply price $PS^{gas}_{\hat{g}t}$ and sells it at the destination node $\tilde{g}$ to final consumers at the final demand price $PD^{gas}_{\tilde{g}t}$. As the gas needs to be transported from $\hat{g}$ to node $\tilde{g}$, the operator needs to decide about the flow on the different pipelines from node g to $\tilde{g}$, $F^{gas}_{g\tilde{g}t}$ and the respective transport services has to be rented at price $PT^{pipe}_{g\tilde{g}t}$. Thus, the maximization problem of the pipeline trade is given as:

$$\max \sum_{\hat{g}gt} \left(PD^{gas}_{gt} - PS^{gas}_{\hat{g}t}\right) T_{\hat{g}gt} - \sum_{g\tilde{g}t} PT^{pipe}_{g\tilde{g}t} F^{gas}_{g\tilde{g}t} \tag{8}$$

$$\text{s.t.} \quad \sum_{\tilde{g}} F^{gas}_{\tilde{g}gt} + \sum_{\tilde{g}} T_{g\tilde{g}t} = \sum_{\tilde{g}} F^{gas}_{g\tilde{g}t} + \sum_{\tilde{g}} T_{\tilde{g}gt} \quad (PN^{gas}_{gt}) \quad \forall g \in \mathcal{G} \tag{9}$$

$$T_{\hat{g}gt}, F^{gas}_{\tilde{g}gt} \geq 0$$

Equation (9) is the flow conservation constraint for pipeline flows which states that at each node incoming and outgoing flows have to be balanced. The associated dual variable is denoted as PN^{gas}_{gt} and can be interpreted as the price of an additional unit of natural gas at that node.

The pipeline operator organizes flows $F^{pipe}_{g\tilde{g}t}$ on a particular pipeline from node g to node $\tilde{g}$ which cause cost $c^{pipe}_{g\tilde{g}t}$. Besides operating the pipelines, the operator also needs to decide about the pipeline capacity $CAP^{pipe}_{g\tilde{g}t}$ by determining the capacity investment $I^{pipe}_{g\tilde{g}t}$. Given the annual cost of pipeline capacity $ci^{pipe}_{g\tilde{g}t}$, the maximization problem of the pipeline operator becomes:

$$\max \sum_{g\tilde{g}t} \left[\left(PT^{pipe}_{g\tilde{g}t} - c^{pipe}_{g\tilde{g}t} \right) F^{pipe}_{g\tilde{g}t} - ci^{pipe}_{g\tilde{g}t} I^{pipe}_{g\tilde{g}t} \right] \tag{10}$$

$$I^{pipe}_{g\tilde{g}t} + \overline{cap^{pipe}_{g\tilde{g}}} \geq F^{pipe}_{g\tilde{g}t} \quad (PC^{pipe}_{g\tilde{g}t}) \qquad \forall g, \tilde{g} \in \mathcal{G}, t \in \mathcal{T}^{gas} \tag{11}$$

$$F^{pipe}_{g\tilde{g}t}, I^{pipe}_{g\tilde{g}t} \geq 0$$

3.2. Electricity Market

In the electricity market model we have three market participants: generators, the transmission system operator and final demand. The transmission system operator is the sole trader in the market, buying electricity from generators at their respective nodes and selling to consumers while accounting for network constraints.

Time periods in the electricity model are denoted by $t \in \mathcal{T}^{ele}$. Each time period is subdivided into load segments $k \in \mathcal{K} := \{k_0, k_1, \cdots, k_K\}$. Nodes in the electricity network are given as $e \in \mathcal{E}$ and lines are denoted by $l \in \mathcal{L} \subseteq \mathcal{E} \times \mathcal{E}$.

Final demand is assumed to be linear for all time periods t and load segments k:

$$DEM^{ele}_{ekt} = a^{ele}_{ekt} + b^{ele}_{ekt} P^{ele}_{ekt} \quad \forall e \in \mathcal{E}, k \in \mathcal{K}, t \in \mathcal{T}^{ele} \tag{12}$$

Power plant technology $i \in \mathcal{I}$ is characterized by the heat efficiency η_{if} where $f \in \mathcal{F}$ denotes the set of fuels. The heat efficiency is assumed to be zero if technology i can not produce with fuel f. Fuel f is also characterized by the carbon content θ_f, the carbon price pe_{ekt}, and the fuel price pf_{fekt}. The generator has to decide about the amount of output X_{iekt} and investment into installed capacity CAP^{ele}_{iekt} at cost ci^{ele}_{iekt}:

$$\max \sum_{i,e,k,t} \left[\left(P^{ele}_{ekt} - \sum_{f \text{ if } \eta_{if} > 0} \frac{pf_{fekt} + \theta_f pe_{ekt}}{\eta_{if}} \right) X_{iekt} - ci^{ele}_{iekt} CAP^{ele}_{iekt} \right] \tag{13}$$

$$\text{s.t. } CAP^{ele}_{iekt} \geq X_{iekt} \quad \forall i \in \mathcal{I}, e \in \mathcal{E}, k \in \mathcal{K}, t \in \mathcal{T}^{ele} \quad (PC^{ele}_i) \tag{14}$$

$$X_{iekt}, CAP^{ele}_{iekt} \geq 0$$

In contrast to natural gas flows on pipelines which can be seen as directly controllable, flows on an electricity transmission grid depend on the injection at the different flows and the characteristics of the network. In particular, flows on the network are determined by the thermal capacity of single lines. Thus, investing into transmission line capacity alters the calculation of the power flows along the network. The physical properties of the electricity network are described by the arc node incidence matrix i_{le}, the line reactance x_l, and the benchmark line capacity given as $\overline{cap^{line}_{lt}}$. The network operator decides about net-injection into the grid at node e, Y^{ele}_{ekt} and the line capacity CAP^{line}_{lt}

given the electricity price and the investment cost ci^{line}_{lt}. Given the decisions, the voltage angle difference Δ_e and the flow on line l, F^{ele}_{lkt} are determined by physical laws:[4]

$$\max \sum_{e,k,t} P^{ele}_{ekt} Y^{ele}_{ekt} - \sum_{l,t} ci^{line}_{lt} CAP^{line}_{lt} \tag{15}$$

$$\text{s.t. } CAP^{line}_{lt} + \overline{cap^{line}_{lt}} \geq |F^{ele}_{lkt}| \quad \forall l \in \mathcal{L}, k \in \mathcal{K}, t \in \mathcal{T} \tag{16}$$

$$\frac{CAP^{line}_{lt} + \overline{cap^{line}_{lt}}}{\overline{cap^{line}_{lt}}} \frac{1}{x_l} \sum_e i_{le} \Delta_e = F^{ele}_{lkt} \quad (\lambda^F_{lkt}) \quad \forall l \in \mathcal{L}, k \in \mathcal{K}, t \in \mathcal{T} \tag{17}$$

$$Y^{ele}_{ekt} = \sum_l i_{le} F^{ele}_{lkt} \quad (\lambda^Y_{ekt}) \quad \forall e \in \mathcal{E}, k \in \mathcal{K}, t \in \mathcal{T} \tag{18}$$

$$F^{ele}_{lkt}, CAP^{line}_{lt} \geq 0$$

$$Y^{ele}_{ekt}, \Delta_e \text{ free}$$

Equation (18) implicitly defines the flow on a line based on the net injection at nodes e while equation (17) determines the linkage between capacity investments and power flows. Equation (16) restricts the flow on electricity line l by the install transmission capacity. As transmission lines are modeled as undirected arcs, the equation holds for either direction and the corresponding multipliers are denoted as PC^{Line+}_{lkt} and PC^{Line-}_{lkt}.

3.3. Market Clearing Conditions

The market clearing condition for electricity equilibrates net-injection into the electricity grid to electricity generation net of final demand and storage. Using the perpendicular sign ($\perp$) to denote complementarity of market clearing and the respective price, the electricity market clearing condition becomes:

$$X_{ekt} = DEM^{ele}_{ekt} + Y_{ekt} \quad \perp \quad P^{ele}_{ekt} \text{ free} \quad \forall e \in \mathcal{E}, k \in \mathcal{K}, t \in \mathcal{T}^{ele} \tag{19}$$

On the natural gas supply market, gas extractors sell gas to natural gas traders:

$$X^{gas}_{gt} \geq \sum_{\tilde{g}} T_{g\tilde{g}t} \quad \perp \quad PS^{gas}_{gt} \geq 0 \quad \forall g \in \mathcal{G}, t \in \mathcal{T}^{gas} \tag{20}$$

Natural gas trades have to buy pipeline transport services offered by pipeline operators:

$$F^{pipe}_{pt} \geq \sum_{\hat{g}} F^{gas}_{\hat{g}pt} \quad \perp \quad PT^{pipe}_{pt} \geq 0 \quad \forall\forall p \in \mathcal{P}, t \in \mathcal{T}^{gas} \tag{21}$$

The two energy markets, natural gas and electricity, are coupled using the market clearing equation for natural gas final demand. In order to establish this link, we need

[4] A more detailed explanation of the line flows and its dependency on the amount invested into the grid is given in Appendix B

to establish a mapping from electricity to natural gas network nodes. We assume, that each electricity node can be served by only one natural gas node but a natural gas node can serve multiple electricity nodes. This mapping is denoted by $\mathcal{M}_{eg}^{N} \subset \mathcal{E} \times \mathcal{G}$.

Besides the locational mapping, we also need to establish a temporal matching as the market may operate at different time scales. We assume, that natural gas prices are constant within a period of the electricity model, i.e. the gas price does not vary across load segments. Furthermore, it is assumed that each electricity period belongs to exactly one natural gas model period. However, one natural gas model period may serve several electricity periods. We denote this mapping by $\mathcal{M}_{t^{ele}t^{gas}}^{T} \subset \mathcal{T}^{ele} \times \mathcal{T}^{gas}$. Given these mappings the natural gas market clearing equation becomes:

$$\sum_{\hat{g}} T_{\hat{g}gt^{gas}} \geq DEM_{gt^{gas}}^{gas} + \sum_{\substack{e \in \mathcal{M}_{eg}^{N} \\ k,t^{ele} \in M_{t^{ele}t^{gas}}^{T} \\ i \text{ if } \eta_{i'gas'} > 0}} \frac{X_{iekt^{ele}}}{\eta_{i'gas'}} \quad \perp \quad PD_{gt^{gas}}^{gas} \geq 0 \quad \forall g \in \mathcal{G}, t \in \mathcal{T}^{gas} \tag{22}$$

On the left hand side of equation (22) the supply at node g is given as the sum of all source node $\hat{g}$ delivering to that node. On the right hand side total demand is given as the sum of final and electricity demand. Electricity demand is derived by using the mapping between the network nodes, summing out the electricity time periods, and identifying gas demanding technologies by a positive heat efficiency for natural gas.

4. Scenarios

4.1. Numerical Illustration

Following, we will provide simple numerical test cases to show the basic functionality of the developed model and highlight the interaction of electricity and natural gas investments. Starting with a simple linear design the substitution effect can be assessed without the impact of loop flows. The linear test case consists of four electricity nodes connected with three lines (Figure 2, left panel). Three nodes have generation capacities and the last node has a fixed demand. The generation capacities are ranked by costs with the cheapest plant having the longest distance from demand. The overall line capacities as well as installed generation capacities are insufficient to cover the full demand level. The three electricity generation nodes are mirrored by a symmetric natural gas network without capacity limitations. However, in the initial setting there is no pipeline connecting the demand node. Given this simple setting it is obvious that the cheap generation option has the highest need for network investments to satisfy the demand level. Therefore, depending on the underlying cost levels the cheap generator will be the primary investment option as long as network extension costs are lower than the cost disadvantage of the next costly generator and so forth. Similar, the option to invest into extension of the gas network and construction of a generator directly at the demand node will be chosen if the costs of pipeline extension and subsequent pipeline transport costs are lower than extending the last electricity line.

The right panel of Figure 2 shows this setup for an arbitrary numerical parametrization. We derive the optimal investment pattern different electricity transmission extension costs while both the generation and pipeline investment costs are fixed. The transmission cost range between 0% and 200% of the generator investment costs. Depending on the cost ratio different extension patterns are optimal. The expansion is focused on the cheap unit and a respective extension of the full network (not shown in Figure 2) for low transmission extension costs, on the expensive unit for medium costs, and finally the natural gas network is extended and generation capacity is built directly at the demand node for high electricity transmission extension costs.

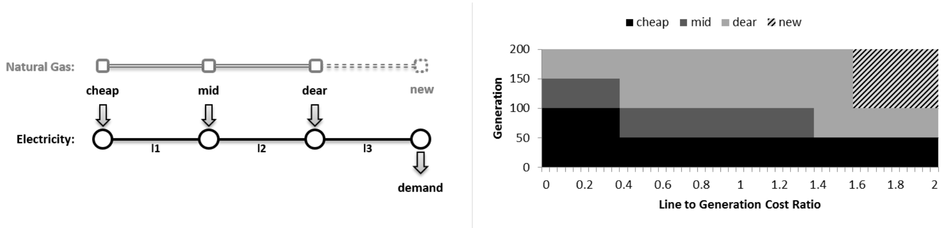

Figure 2: Linear Test Case

The linear setup could be interpreted as a representation of a model formulation that treats electricity as a directed and controllable flow that accounts for transmission limitations but not for power flow characteristics. One advantage of the proposed model formulation is its capability to deal with meshed electricity networks and the resulting loop flow impacts. Therefore, the second test case extends the linear setup by introducing a meshed electricity network topology (Figure 3, left panel).

In Figure 3 we extend the network with two auxiliary nodes vis-à-vis the mid and dear generation nodes and additional lines connecting the new nodes with the existing ones. The transmission capability is adjusted to allow the same initial transmission as in the linear case. The remainder of the system is kept identical.

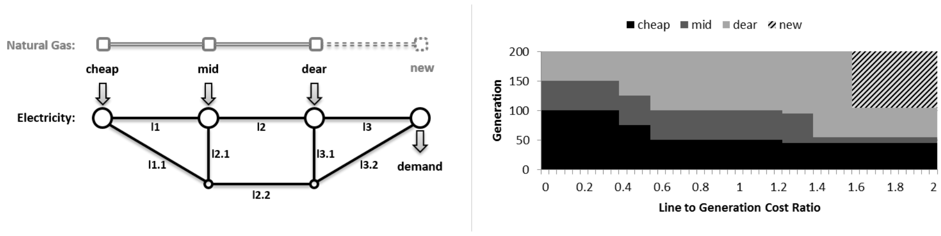

Figure 3: Meshed Test Case

We perform the same extension analysis by varying transmission investment costs. Although, the meshed network has a high similarity to the linear case the resulting extension pattern slightly differs (Figure 3, right panel). Firstly, there are two additional extension phases and secondly, the final natural gas extension is slightly lower as more generation from the mid prices generator is utilized compared to the linear setting. Naturally, those differences are based on the chosen network topology and the underlying cost parameter and cannot be generalized. Nevertheless, the example shows that due to the physical characteristics of electricity transmission the resulting extension pattern can easily deviate from a simplified linear representation even for highly simplistic network setups.

The numerical test cases provide us with three basic insights. First, the developed model is in principle capable of simulating combined electricity and natural gas market settings and provides results that are in line with expected outcomes. Second, the substitution effect between gas and electricity remains valid for both simplified linear settings and meshed networks, although the actual extension strongly depends on the underlying cost parameters. And third, the nature of meshed networks makes clear predictions of optimal investments more complicated and requires the need for subsequent modeling with power flow elements. A simplified linear treatment neglecting loop flow externalities is likely to lead to wrong estimates.

4.2. Stylized Application to European Networks

We will now turn to analyze the European electricity and natural gas markets with a stylized numerical model to evaluate the potential impact of the substitution effect under real world market conditions. The analysis is focused on continental Central Europe (Figure 4). Each country is linked with its neighboring countries via aggregated connections representing 220kV and 380kV transmission lines respectively for the electricity network (ENTSO-E, 2013) and cross-border pipelines in the natural gas network (ENTSOG, 2013). The respective connection length is derived as geographic distance between the country nodes. Furthermore, in the natural gas network the main import options (Russia, Africa, and LNG from the Atlantic and Mediterranean) are connected with the relevant European country nodes.[5]

Each node has the aggregated country's electricity generation plant capacity clustered into ten types following ENTSO-E (2013) with average plant efficiency values and capacity values. Demand is derived from the hourly load values as published by ENTSO-E and adjusted to match aggregated yearly demand with the values provided in ENTSO-E (2013). Three load segments are used – peak, mid, and off-peak – which are derived by ordering hourly load according to the total European demand level and taking the average values for each third, respectively. Natural gas demand is taken from OECD (2013) with natural gas production for European countries taken from Eurostat. The non-European gas producers are assumed to have unlimited production capacities. Their export potential is limited by the pipeline capacities towards Europe. Fuel prices are based on 2012 values taken from BAFA for German import prices and adjusted to 90% for East European countries and 110% for South European countries. The production costs of the non-European gas producers are calibrated to derive a similar natural gas price level in the model as provided by BAFA (ca. 30 €/MWh).

[5]LNG import capacities are treated like pipeline capacity restrictions on the respective connection.

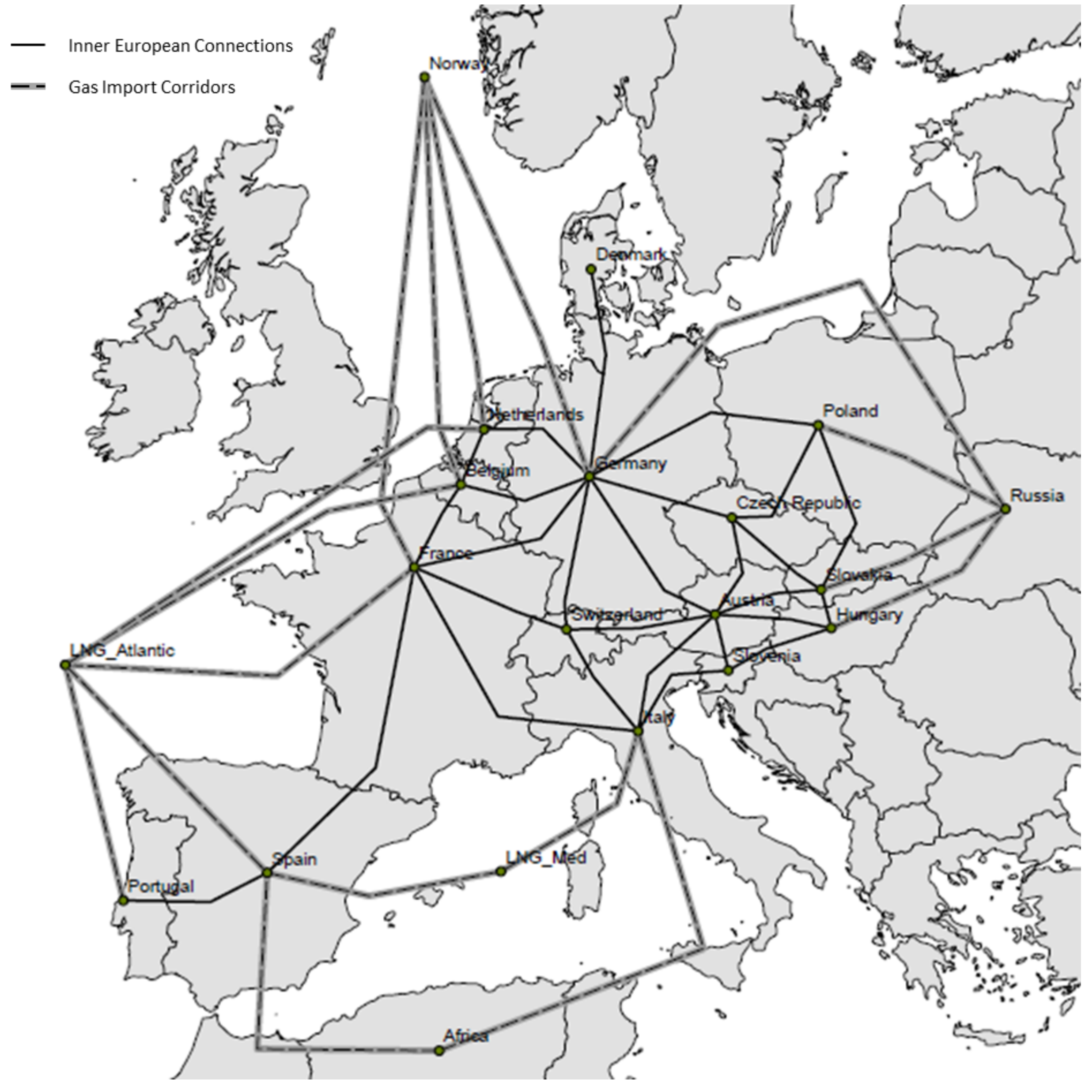

Figure 4: Stylized European Network Representation

Although, simplified in nature the model provides reasonable market results for a 2012 benchmark simulation. The natural gas market shows lower prices in East Europe and higher prices in South Europe. The LNG supply option plays a crucial role in supplying the Iberian markets while Central Europe is supplied by endogenous production and Norwegian and Russian imports. The electricity market shows higher prices in countries that depend on gas production like Southern Europe and the Benelux. Prices during off-peak are on average about 70% of the medium price level while peak prices represent about 110%. The lower markup between the mid and peak segment is driven by the dominance of gas plants as marginal units in both load segments in many European countries. Congestion occurs mainly on lines towards Italy as well as partially on lines from France towards Belgium and Spain.

In order to test whether the European markets are subject to potential investment substitution effects we derive a scenario setting that provides incentives to extend both natural gas generation and transmission lines. Given the current objectives of European energy policy it is expected that firstly, coal fired generation will be penalized due to its higher CO_2 emissions and secondly, renewable generation will further increase its production share. To capture those two elements we introduce a price markup for coal based generation effectively pushing it out of the market and we double the renewable generation share. This will in turn lead to more investments into gas fired generation and subsequent investments in transmission lines and/or natural gas infrastructure.

Transmission investment costs are based on L'Abbate and Migliavacca (2011) with an assumed average of 500k€ per km for a 380kV line. Natural gas pipeline costs are based on INGAA (2009) with an assumed average of 1Mio€ per km for a 32 inch pipeline. Finally, gas fired combined cycle investment costs are based in EIA (2013) with an assumed average of 650 € per MW. For all investment costs the annuity is derived using a lifetime of 20 years and an interest rate of 4% for the network investments and 7% for the plant investment. As the substitution between those investments is largely determined by the underlying investment costs we vary the electricity transmission investment cost by 50% upward and downward. Power plant and pipeline investment costs as well as all other parameters are kept constant.

Table 1 provides an overview about the carried out investments in the three cost scenarios. While it is not surprising to see significant higher transmission investment if the extensions costs are lower the pattern over all three investment options shows a clear indication for the substitution between gas and electricity infrastructure. The most obvious example is the Italian situation. In case of cheap transmission extension costs Italian electricity demand will be satisfied by imports from North and East Europe with Switzerland and Austria extending their cross-border capacities respectively. In case of high transmission extension cost Italian demand is supplied by extending the gas pipelines towards Italy and Slovenia and constructing new gas fired plants in those countries thereby avoiding electricity transmission investments altogether. A reversed impact can be observed in Poland. Due to the model calibration Poland faces low gas prices thanks to its large supply of Russian gas. The low price coupled with the high dependence of coal in the original setting makes its both profitable and necessary to invest into new gas plants in Poland. The generated electricity is then distributed towards the demand in West and South Europe. However, with increasing transmission costs this incentive is greatly reduced and consequently investments in gas plants in Poland decline while the plants are constructed in the original electricity import countries (i.e. Germany, Czech Republic).

Although the results of the stylized model are to be taken with care in respect to actual developments in Europe they highlight the strong interdependence of natural gas and electricity markets. The seemingly high difference in investment costs of 50% to 150% may not be too unrealistic given the large share of delayed investments in energy infrastructure (i.e. due to NIMBY or regulatory induced longer planning phases). Coupled with further changes on the overall market conditions (i.e. the impact of US

Table 1: Investment Scenario Results

Costs Scenario	**50%**	**100%**	**150%**
Electricity Transmission Investment [GW]			
Spain - France	10.81	0.97	-
Germany - Poland	1.3	-	-
Poland - Czech Republic	5.01	3.51	-
Austria - Hungary	5.14	-	-
Austria - Italy	10.54	3.47	0.03
Switzerland - Italy	1.89	3.36	-
Natural Gas Pipeline Investment [GW]			
Austria - Italy	6.66	12.85	19.95
Hungary - Slovenia	2.14	2.14	4.88
Gas Power Plant Investment [GW]			
Austria	-	5.14	-
Czech Republic	-	-	1.79
Denmark	1.09	1.95	1.95
Germany	-	-	1.46
Hungary	2.78	0.32	-
Italy	-	-	2.08
Netherlands	4.89	9.06	9.36
Poland	24.54	21.38	19.65
Slovakia	1.07	3.41	3.49
Slovenia	-	-	2.15

shale gas on natural gas prices in Europe) the production and investment cost ratios can easily fluctuate and thereby change the optimal investment pattern in both markets. Given the expectation that natural gas will play a major role in transferring electricity markets towards a high share of fluctuating renewables decision where and when to build and extend gas capacities should not be considered in a singular electricity market perspective.

5. Conclusion

This paper has analyzed the interaction between natural gas and electricity investments while accounting for the network characteristics of both markets. The model is formulated as a partial equilibrium representation using the MCP format. Albeit, the presented model is limited to production and transport aspects the formulation allows an easy extension to capture further market actors like LNG traders or the dynamic nature of storage operation. The chosen model framework also enables an easy adoption of different market designs, i.e. oligopolistic competition on the production and generation markets and regulated decisions on the network stage. Applying the model to a four node test case and a stylized representation of the continental European natural gas and electricity markets, we can present two important insights for future market evaluations. First, the meshed nature of electricity transmission and power flows leads to a high complexity that needs to be captured in investment models to provide reasonable evaluations. This issue is techno-economic in nature and requires the inclusion of basic electrical-engineering elements in market models. A simplified representation of power flows via pure (directed) trade flows is likely to provide biased results. Second, natural gas and electricity markets face a mutual interdependence in investment decision that requires a combined approach to be adequately captured within model estimates. As optimal investment decision of substituting alternatives (i.e. pipeline and power plant investments vs. transmission line and plant investments) depend on the locational price spreads in the markets which in turn strongly depend on the chosen investments an integrated assessment is needed. Capturing this interaction in separated models is likely to provide biased results. The investment substitution aspect is furthermore amplified by the problems of meshed electricity networks. The paper provides further insights for the ongoing discussion about the future development about market design in gas and electricity markets. Most of this discussion is focused on single aspects – i.e. the capacity market debate in electricity generation or the question about optimal network regulation to foster optimal investments – although they are finally interlinked and therefore warrant a comprehensive and encompassing approach. The developed model can help to provide such a bridge between the ongoing fields and help to develop robust market and policy recommendations for the challenges at hand.

References

Abrell, J. and Weigt, H. (2012): Combining Energy Networks. Networks and Spatial Economics, *Network and Spatial Economics* 12 (3):377–401.

An, S., Li, Q., and Gedra, T.W. (2003): Natural gas and electricity optimal power flow, In: *Transmission and Distribution Conference and Exposition*, Volume 1, pp. 138–143, IEEE PES.

Bakken, B. H., Skjelbred, H. I., and Wolfgang, O. (2007): eTransport: Investment planning in energy supply systems with multiple energy carriers, *Energy* 9:1676–1689.

Brooke, A., Kendrick, D., and Meeraus, A. (2008): *GAMS A User's Guide*, GAMS Development Cooperation.

Damavandi, M. Y., Kiaei, I., Sheikh-El-Eslami, M. K., and Seifi, H. (2011): New approach to gas network modeling in unit commitment, *Energy* 36 (10):6243–6250.

Ferris, M. C. and Munson, T. S. (2000): Complementarity Problems in GAMS and the Path Solver, *Complementarity Problems in GAMS and the Path Solver* 24 (2):165–188.

Geidl, M. and Andersson, G. (2005): A modeling and optimization approach for multiple energy carrier power flow, In: *Power Tech*, pp. 1–7, IEEE.

Geidl, M. and Andersson, G. (2006): Operational and structural optimization of multi-carrier energy systems, *European Transactions on Electrical Power* 16 (5):463–477.

Gil, E. M., Quelhas, A. M., McCalley, J. D., and Van Voorhis, T. (2003): Modeling integrated energy transportation networks for analysis of economic efficiency and network interdependencies, In: *North American Power Symposium (NAPS)*.

IEA (2011a): *World Energy Outlook 2011*, International Energy Agency, OECD.

IEA (2011b): *World Energy Outlook 2011. Special Report: Are we Entering a Golden Age of Gas?*, International Energy Agency, OECD.

Koeppel, G. and Andersson, G. (2009): Reliability modeling of multi-carrier energy systems, *Energy* 34 (3):235–244.

Krause, T., Kienzle, F., Liu, Yang, and Andersson, G. (2011): Modeling interconnected national energy systems using an energy hub approach, In: *PowerTech*, pp. 1–7, IEEE.

Li, T., Eremia, M., and Shahidehpour, M. (2008): Interdependency of Natural Gas Network and Power System Security, *IEEE Transactions on Power Systems* 23 (4):1817–1824.

Lienert, M. and Lochner, S. (2012): The importance of market interdependencies in modeling energy systems – The case of the European electricity generation market, *International Journal of Electrical Power & Energy Systems* 34 (1):99–113.

Liu, C., Shahidehpour, M., Fu, Yong, and Li, Zuyi (2009): Security-Constrained Unit Commitment With Natural Gas Transmission Constraints, *IEEE Transactions on Power Systems* 24 (3):1523–1536.

Mello, O. D. and Ohishi, T. (2006): An Integrated Dispatch Model Of Gas Supply And Thermoelectric Generation With Constraints On The Gas Supply, In: *X Symposium Of Specialists In Electric Operational And Expansion Planninatural Gas*.

Quelhas, A. M., Gil, E., and McCalley, J. D. (2006): Nodal prices in an integrated energy system, *International Journal of Critical Infrastructures* 2 (1):50–69.

Rubio, R., Ojeda-Esteybar, D., Ano, O., and Vargas, A. (2008): Integrated natural gas and electricity market: A survey of the state of the art in operation planning and market issues, In: *Transmission and Distribution Conference and Exposition: Latin America*, pp. 1–8, IEEE/PES.

Shahidehpour, M., Fu, Y., and Wiedman, T. (2005): Impact of Natural Gas Infrastructure on Electric Power Systems, *Proceedings of the IEEE* 93 (5):1042–1056.

Unsihuay, C., Lima, J. W. M., and de Souza, A. C. Z. (2007): Modeling the Integrated Natural Gas and Electricity Optimal Power Flow, In: *Power Engineering Society General Meeting*, pp. 1–7, IEEE.

Unsihuay-Vila, C., Marangon-Lima, J.W., de Souza, A.C.Z., Perez-Arriaga, I.J., and Balestrassi, P.P. (2010): A Model to Long-Term, Multiarea, Multistage, and Integrated Expansion Planning of Electricity and Natural Gas Systems, *IEEE Transactions on Power Systems* 25 (2):1154–1168.

Urbina, M. and Li, Z. (2008): Modeling and analyzing the impact of interdependency between natural gas and electricity infrastructures, In: *Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century*, pp. 1–6, IEEE.

Appendix

A. Mixed Complementarity Formulations

A.1. Natural Gas Market Model

For the natural gas model the MCP version is derived by deriving the necessary (and sufficient as the problems are linear) first order conditions of the optimization problems of the natural gas producer (equations 6 and 7), the pipeline trader (equations 8 and 9), and the pipeline operator (equations 10 and 11). The natural gas sub-model is closed by adding a market clearing condition for natural gas at supply (A.10) and demand nodes (A.9) as well as for pipeline transport services (A.11):

$$c^{gas}_{gt} + PC^{gas}_{gt} \geq PS^{gas}_{gt} \quad \perp \quad X^{gas}_{gt} \geq 0 \quad \forall g,t \tag{A.1}$$

$$cap^{gas}_{gt} \geq X^{gas}_{gt} \quad \perp \quad PC^{gas}_{gt} \geq 0 \quad \forall g,t \tag{A.2}$$

$$PS^{gas}_{gt} + PN^{gas}_{\tilde{g}t} \geq PD^{gas}_{\tilde{g}t} + PN^{gas}_{gt} \quad \perp \quad T^{gas}_{g\tilde{g}t} \geq 0 \quad \forall g,\tilde{g},t \tag{A.3}$$

$$PT^{pipe}_{g\tilde{g}t} + PN^{gas}_{gt} \geq PN^{gas}_{\tilde{g}t} \quad \perp \quad F^{gas}_{g\tilde{g}t} \geq 0 \quad \forall g,\tilde{g},t \tag{A.4}$$

$$\sum_{\tilde{g}} F^{gas}_{\tilde{g}gt} + \sum_{\tilde{g}} T_{g\tilde{g}t} = \sum_{\tilde{g}} F^{gas}_{g\tilde{g}t} + \sum_{\tilde{g}} T_{\tilde{g}gt} \quad \perp \quad PN^{gas}_{gt} \geq 0 \quad \forall g,t \tag{A.5}$$

$$c^{pipe}_{g\tilde{g}t} + PC^{pipe}_{g\tilde{g}t} \geq PT^{pipe}_{g\tilde{g}t} \quad \perp \quad F^{pipe}_{g\tilde{g}t} \geq 0 \quad \forall g,\tilde{g},t \tag{A.6}$$

$$ci^{pipe}_{g\tilde{g}t} \geq PC^{pipe}_{g\tilde{g}t} \quad \perp \quad I^{pipe}_{g\tilde{g}t} \geq 0 \quad \forall g,\tilde{g},t \tag{A.7}$$

$$I^{pipe}_{g\tilde{g}t} + \overline{cap^{pipe}_{g\tilde{g}}} \geq F^{pipe}_{g\tilde{g}t} \quad \perp \quad PC^{pipe}_{g\tilde{g}t} \geq 0 \quad \forall g,\tilde{g},t \tag{A.8}$$

$$\sum_{\tilde{g}} T^{gas}_{\tilde{g}gt} \geq a^{gas}_{gt} + b^{gas}_{gt} PD^{gas}_{gt} \quad \perp \quad PD^{gas}_{gt} \geq 0 \quad \forall g,t \tag{A.9}$$

$$X^{gas}_{gt} \geq \sum_{\tilde{g}} T^{gas}_{g\tilde{g}t} \quad \perp \quad PS^{gas}_{gt} \geq 0 \quad \forall g,t \tag{A.10}$$

$$F^{pipe}_{g\tilde{g}t} \geq F^{gas}_{g\tilde{g}t} \quad \perp \quad PT^{pipe}_{g\tilde{g}t} \geq 0 \quad \forall g,\tilde{g}t \tag{A.11}$$

A.2. Electricity Market Model

For the electricity market model the MCP version is derived using the first order conditions of the generators' maximization problem (equations 13 and 14) and the grid operator's maximization problem (equations 15 to 18). The electricity market model is closed by adding the market clearing condition for electricity (19):

$$\sum_{f \text{ if } \eta_{if}>0} \frac{pf_{fekt} + \theta_f pe_{ekt}}{\eta_{if}} + PC^{ele}_{iekt} \geq P^{ele}_{ekt} \qquad \perp \quad X_{iekt} \geq 0 \tag{A.12}$$

$$ci^{ele}_{iekt} \geq PC^{ele}_{iekt} \qquad \perp \quad CAP^{ele}_{iekt} \geq 0 \tag{A.13}$$

$$CAP^{ele}_{iekt} \geq X_{iekt} \qquad \perp \quad PC^{ele}_{iekt} \geq 0 \tag{A.14}$$

$$P^{ele}_{ekt} + \lambda^{Y}_{ekt} = 0 \qquad \perp \quad Y^{ele}_{ekt} \text{ free} \tag{A.15}$$

$$\sum_k \left(PC^{Line+}_{lt} + PC^{Line-}_{lt}\right) + \sum_k \left(\lambda^F_{lkt} \frac{\sum_e i_{le}\Delta_e}{x_l \overline{cap^{line}_{lt}}}\right) \leq ci^{line}_{lt} \qquad \perp \quad CAP^{line}_{lt} \geq 0 \tag{A.16}$$

$$PC^{Line+}_{lkt} - PC^{Line-}_{lkt} \geq \sum_e i_{le}\lambda^Y_{ekt} - \lambda^F_{ekt} \qquad \perp \quad F_{lkt} \geq 0 \tag{A.17}$$

$$\sum_l \lambda^F_{lkt} \frac{CAP^{line}_{lt} + \overline{cap^{line}_{lt}}}{\overline{cap^{line}_{lt}}} \frac{1}{x_l} \sum_e i_{le} = 0 \qquad \perp \quad \Delta_l \text{ free} \tag{A.18}$$

$$CAP^{line}_{lt} + \overline{cap^{line}_{lt}} \geq F^{ele}_{lkt} \qquad \perp \quad PC^{Line+}_{lkt} \geq 0 \tag{A.19}$$

$$F^{ele}_{lkt} \geq CAP^{line}_{lt} + \overline{cap^{line}_{lt}} \qquad \perp \quad PC^{Line-}_{lkt} \geq 0 \tag{A.20}$$

$$\frac{CAP^{line}_{lt} + \overline{cap^{line}_{lt}}}{\overline{cap^{line}_{lt}}} \frac{1}{x_l} \sum_e i_{le}\Delta_e = F^{ele}_{lkt} \qquad \perp \quad \lambda^F_{lkt} \text{ free} \tag{A.21}$$

$$Y^{ele}_{ekt} = \sum_l i_{le} F^{ele}_{lkt} \qquad \perp \quad \lambda^Y_{ekt} \text{ free} \tag{A.22}$$

$$X_{ekt} = DEM^{ele}_{ekt} + Y_{ekt} \qquad \perp \quad P^{ele}_{ekt} \text{ free} \tag{A.23}$$

B. Explanation of the electricity line flow equations

Following the DC-Load Flow approach power flow F on a line l can be derived using the voltage angel difference Δ between the connected nodes. Assuming that the line's resistance is significant smaller than the line's reactance x_l the flow can be expressed as follows:

$$F_l = \frac{1}{x_l}\Delta_l \tag{B.1}$$

The extension of a given line can be considered as adding a second parallel circuit on the connection with a specific reactance value tied to the chosen capacity extension. Following the law of parallel circuits the total reactance of a line with several parallel circuits n can be expressed as follows:

$$\frac{1}{x_l} = \sum_n \frac{1}{x_n} \tag{B.2}$$

With X_n as the individual reactances of the different circuits composing the line l. If the line consists of N identical parallel circuits the expression can be simplified to:

$$\frac{1}{x_l} = N\frac{1}{x_n} \tag{B.3}$$

Applied to the logic of line extensions: adding a second identical line to an existing connection leads to a bisection of the initial reactance. Therefore, given an initial system with starting line capacities $\overline{cap_l^{line}}$ and respective line reactances $\overline{x}_l$ the impact of a line extension can be formulated as:

$$\frac{1}{x_l} = \frac{CAP_l^{line} + \overline{cap_l^{line}}}{\overline{cap_l^{line}}}\frac{1}{x_n} \tag{B.4}$$

This formulation is naturally an approximation as line extensions are typically integer decisions and capacity and reactance don't need to be in a fixed relation. It also requires an initial system and does only allow extension of existing connection but no completely new connections. Furthermore, the decommissioning of a line is not possible, as this would require x_l to become infinite. Equation (17) in the main text results by substituting (B.4) back into equation (B.1) and accounting for nodal based representation of the voltage angle ($\Delta_l = \sum_e i_{le}\Delta_e$).

C. Nomenclature

Sets

$\mathcal{M}_{eg}^{N} \subset \mathcal{E} \times \mathcal{G}$	Mapping from electricity to natural gas nodes.
$\mathcal{M}_{t^{ele}t^{gas}}^{T} \subset \mathcal{T}^{ele} \times \mathcal{T}^{gas}$	Mapping from electricity to natural gas model periods.
$e \in \mathcal{E}$	Nodes in electricity network
$g \in \mathcal{G}$	Nodes in natural gas network
$i \in \mathcal{I}$	Power plant technologies
$k \in \mathcal{K}$	Load segments per time period in electricity model.
$l \in \mathcal{L}$	Electricity transmission lines
$t \in \mathcal{T}^{ele}$	Time periods electricity model
$t \in \mathcal{T}^{gas}$	Time periods in natural gas model

Parameters

η_{if}	Heat efficiency of technology i using fuel f
$\overline{cap_{lt}^{line}}$	Reference capacity of electricity line l
$\overline{cap_{g\tilde{g}}^{pipe}}$	Exogenous initially given pipeline capacity from node g to $\tilde{g}$
θ_f	Carbon content of fuel f
a_{ekt}^{ele}	Intercept demand function electricity at node e in period t and load segment k
a_{gt}^{gas}	Intercept linear demand natural gas at node g in period t

b_{ekt}	Slope demand function electricity at node e in period t and load segment k
b^{gas}_{gt}	Slope linear demand function natural gas at node g in period t
c^{gas}_{gt}	Cost natural gas extraction at node g in period t
$c^{pipe}_{g\tilde{g}t}$	Cost for natural gas flow Flow along the pipeline from node g to $\tilde{g}$ in period t
cap^{gas}_{gt}	Capacity natural gas extraction at node g in period t
$cap^{pipe}_{g\tilde{g}}$	Capacity pipeline from node g to node $\tilde{g}$
cap^{liq}_{g}	Capacity for LNG liquefaction at node g
cap^{reg}_{g}	Capacity for LNG regasification at node g
ci^{ele}_{iekt}	Cost of investment into installed capacity of type i at node e in period t and load segment k
ci^{line}_{lt}	Cost investment into line l in period t
$ci^{pipe}_{g\tilde{g}t}$	Cost investment into pipeline capacity from node g to $\tilde{g}$ in period t
i_{le}	Arc-node incidence matrix for electricity network. I.e. becomes one if l starts in node e, minus one if ends there, and zero if no connection exists
pe_{ekt}	Price carbon at node e in period t load segment k.
pf_{fekt}	Price of fuel f at node e in period t and load segment k
x_l	Reactance of electricity line l

Variables

Δ_e	Voltage angle difference at node e compared to the slack node
λ^F_{lkt}	Lagrangian multiplier on voltage angle equation
λ^Y_{ekt}	Lagrangian multiplier line flow equation
CAP^{ele}_{iekt}	Investment into installed capacity of type i at node e in period t and load segment k
CAP^{line}_{lt}	Investment into electricity line l in period t
DEM^{gas}_{gt}	Natural gas demand at node g in period t
DEM^{ele}_{ekt}	Demand electricity at node e in period t and load segment k
$F^{gas}_{g\tilde{g}t}$	Flow of natural gas along the pipeline from node g to $\tilde{g}$ in period t
$F^{pipe}_{g\tilde{g}t}$	Flow along the pipeline from node g to $\tilde{g}$ in period t
F^{ele}_{lkt}	Electricity flow on line l in period t and load segment k
$I^{pipe}_{g\tilde{g}t}$	Investment into pipeline capacity from node g to $\tilde{g}$ in period t
P^{ele}_{ekt}	Electricity price at node e in period t and load segment k
PC^{ele}_{iekt}	Scarcity price generation capacity for plant i at node e in period t load segment k
PC^{gas}_{gt}	Scarcity price of natural gas extraction capacity Natural at node g in period t
PC^{Line+}_{lkt}	Scarcity rent on electricity line capacity rent positive direction

PC^{Line-}_{lkt}	Scarcity rent on electricity line capacity rent negative direction
$PC^{pipe}_{g\tilde{g}t}$	Scarcity price pipeline capacity from node g to $\tilde{g}$ in period t
PD^{gas}_{gt}	Final demand price gas at node g in period t
PN^{gas}_{gt}	Nodal price of natural gas at node n in period t
PS^{gas}_{gt}	Natural gas supply price at node g in period t
$PT^{pipe}_{g\tilde{g}}$	Transport price for natural gas along the pipeline from node g to $\tilde{g}$ in period t
X^{gas}_{gt}	Natural gas extraction at node g in period t
X_{iekt}	Output of plant technology i at node e in period t and load segment k
Y^{ele}_{ekt}	Net electricity injection into the grid at node e in period t and load segment k