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feature article

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In a dispute with more than just scientific import, Alexis Clairaut, Leonhard Euler, and Jean le Rond d'Alembert each employed their own strategies to establish that they were the first to understand a puzzling feature of the Moon's orbit.

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About two years after *Sputnik 1* began emitting radio signals, the Soviet Union's *Luna 2* became the first manmade object to reach the Moon. The intentional crash landing of the spacecraft on 13 September 1959 once again demonstrated Soviet technological superiority over the US. That event can be regarded as the trigger of the so-called race to the Moon, initiated by President John F. Kennedy in May 1961. As the much-broadcast commemoration of *Apollo 11* and the first manned landing on the Moon reminded us, the Americans won the competition.

Every story has a prologue. The one preceding Neil Armstrong's historic steps shows that a captivating plot from the past needn't include a cold war or spies. It tells of three men who competed to develop a predictive theory that would accurately describe the motion of the Moon and therefore furnish a key tool that, much later, would help the Soviets and the Americans anticipate the position of their celestial target. The mid-18th-century controversy concerning lunar theory involved strong personal and political interests and shows that even in mathematics, truth isn't always the result of a solely objective and logical demonstration but can be a construct born of tactics, strategy, and intrigue.

The principal actor

The curtain rises on our historical drama. The setting is Paris, capital of France and French science. The calendar reads 15 November 1747, and one of the protagonists of the story about to unfold is leaving his apartment on Berry Street. The 34-year-old mathematician Alexis Claude Clairaut (figure 1) can already look back at an impressive list of achievements, one of which occurred during the expedition to Lapland that had taken place 10 years earlier under the direction of Pierre Louis Moreau de Maupertuis. Despite the plague of flies, the cold winter, and all the dangers of the undertaking—including the perilous transport of astronomical instruments through rapids and dense woods—Clairaut helped prove that Earth is flattened at the poles by measuring a degree of the meridian. As historian Mary Terrall explores in her imposing book *The Man Who Flattened the Earth: Maupertuis and the Sciences in the Enlightenment* (University of Chicago Press, 2002), Clairaut and Maupertuis thus showed that Isaac Newton was right about Earth's oblateness; in contrast, René Descartes's followers had deduced from Descartes's vortex theory of planetary motion and other works that the terrestrial globe was flattened at the equator.

On this autumn day, Clairaut carries under his arm a treatise that criticizes Newton's law of attraction. If Clairaut is well aware of the importance of his assertions, he might still not expect them to start a prominent controversy with two of the most renowned mathematicians of his time: Leonhard Euler and Jean le Rond d'Alembert.

Clairaut couldn't have chosen a better day to present his paper, as that Wednesday in 1747 is the official start of the academic year. On that day, the assembly of the Academy of Sciences in Paris is open to the public, and the meeting room is almost always full. Those special circumstances, in addition to the implications of the talk, might explain the excitement generated by Clairaut's paper in the course of the following months. Historians cannot

Figure 1. Alexis Claude Clairaut (1713–65). This engraving, circa 1770, is by Charles-Nicolas Cochin and Louis Jacques Cathelin, based on a drawing by Louis de Carmontelle.

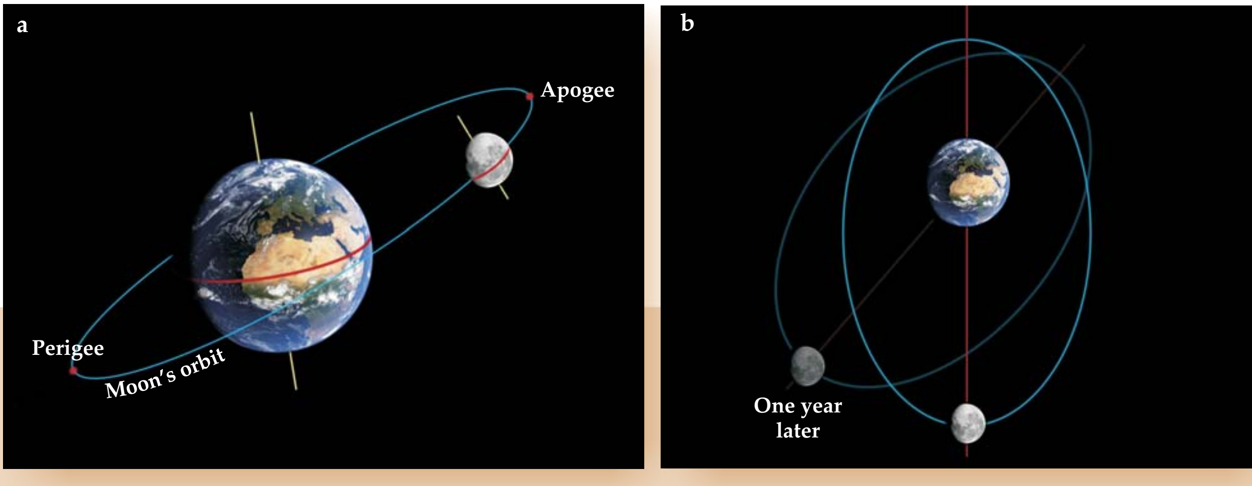


Figure 2. The Moon's orbital motion. (a) The apogee and perigee are key markers of the lunar orbit. Collectively, the two points are called apsides. **(b)** The orbit and apsides rotate at a rate of about 40° per year. The eccentricity of the orbit is exaggerated in the illustrations.

reconstruct exactly how many people came to the meeting; the secretary of the academy, Jean-Paul Grandjean de Fouchy, didn't establish a list of all attendees as he would have done for a typical assembly. But a transcript of Clairaut's talk, inserted in the academy records, allows one to closely follow his thoughts concerning the inverse-square law and the lunar orbit.

Because the Moon is attracted to both Earth and the Sun, mathematicians calculating its orbit must confront the famously difficult three-body problem. Thanks to the Leibnizian calculus and its improvement by the brothers Jakob and Johann Bernoulli, by 1740 the problem could be formulated as a system of four differential equations. But those only allow an approximate solution. One of the numerous irregularities in the orbit of the Moon around Earth is the motion of the apsides—that is, the apogee, or point at which the Moon is farthest from Earth, and the perigee, at which it is nearest. Those two points rotate around Earth with a period of nine years, as illustrated in figure 2. Even the great Newton could account for only half of the motion of the apsides. That's why Clairaut takes on the lunar orbit and tries to explain it with the inverse-square law. But after calculating over and over again, he is forced to the conclusion that the law of gravitation cannot account for the observations—he keeps finding an 18-year period for a full revolution of the lunar apsides around Earth. Clairaut then experiments with adding a $1/r^3$ term to Newton's attraction law to explain the apsidal motion.

To be sure, it isn't the first time that a modification of the attraction law has been proposed. For example, Gabrielle Émilie Le Tonnelier de Breteuil, marquise du Châtelet, had already postulated a similar addition in her *Institutions de physique* of 1740. However, she was concerned with terrestrial events such as the motion of sap in plants or of fluids in capillary tubes, not with astronomical phenomena. Moreover, according to Swiss mathematician Gabriel Cramer and other contemporaries, Clairaut, who was a mentor to the marquise, may have suggested the idea to her. In any event, Clairaut's revolutionary and unorthodox move is to question Newton's law in both the astronomical and terrestrial domains.

For some readers at the time, it seems clear that Clairaut's additional term, specifically introduced to explain the motion of the Moon's apsides, implies that the laws that rule the motion of Earth around the Sun, the fall of an apple to the ground, and the orbit of the Moon are no longer one and the same. That is one of the chief criticisms offered by George-Louis Leclerc, comte de Buffon, author of the famous multivolume *Natural History*. But as science historian John Greenberg points out in a March 1986 *Isis* article, Buffon "defended the inverse-square law as sacred on metaphysical grounds. He harped on the matter *ad nauseum* but contributed nothing to the mathematical work that led to the achievement that eluded Newton." In fact, a thorough reading of Clairaut's paper shows that the French mathematician does not contest the universality of the gravity law. Rather, he demonstrates that the term he adds to correct Newton's formula makes a huge difference for astronomical objects, like the Moon and Earth, attracting at a small distance, but it nearly vanishes for objects, like Earth and the Sun, attracting at a great distance. He thus interprets Newton's inverse-square law as a special case in astronomy and replaces it with what he thinks is a new, universally valid principle.

Dispersed via letters and periodicals, Clairaut's thesis soon reaches other 18th-century science centers such as London, Berlin, and Geneva. In January 1748, for example, the Jesuit *Journal de Trévoux* publishes a review of Clairaut's paper along with a defense of his Newtonian views—the review supposedly initiated by the French scholar himself as a public answer to Buffon's attacks.

Two other protagonists

The drama's first supporting role is played by Swiss mathematician Euler (figure 3). By 1748, then in Berlin, he has already been informed of Clairaut's new term by Clairaut himself, who sent letters dated 3 September and 11 September 1747. The French scholar, though, did not send any mathematical justifications.

Euler's answer to Clairaut might have offended its recipient a bit. Euler agrees that the Newtonian attraction law doesn't seem to work for the Moon but claims that he already pointed out the insufficiency of Newton's law in a yet unpublished treatise on Saturn's motion. He furthermore asserts that he already recognized the problem in his own studies of the motion of the Moon's apsides. It is hard to say for sure when Euler reached that realization. Numerous documents confirm that he had indeed reflected on apsidal motion, even as far back as 1725. Moreover, in a paper read before the Berlin Academy of Science in June 1747, Euler explicitly men-

Figure 3. Leonhard Euler (1707–83). This pastel was created by Jakob Emanuel Handmann in 1753.

tions the need to correct the inverse-square law to account for the motion of the apsides. That paper, however, won't be published until 1749, long after Clairaut presents his revolutionary treatise in Paris.

Furthermore, in his reply to Clairaut, Euler also rejects the universality of the new gravity law proposed by the French mathematician. He argues that in the case of Mercury, whose orbit is close to the Sun, the added term would become too large to accord with the astronomical observations. From that point, the correspondence between the two men becomes more and more contentious. There are many reasons for the increasing friction. For one, Clairaut, who is already being criticized from all parts of Europe, is forced to justify himself and attacks Euler on several mathematical points without always remaining fair. Moreover, although Clairaut informs Euler in April 1748 that Euler's essay on Saturn won a prize given by the Paris Academy of Sciences, he also writes in the same letter that the question deserves further research and should be asked again. Thus Clairaut insinuates that Euler's paper was mediocre and won only because nothing better reached the academy.

Unlike Clairaut and Euler, d'Alembert (figure 4) first became interested in astronomical problems and their mathematical solution in 1746. In that year d'Alembert, the third major player in the drama, begins writing a paper devoted to general astronomical subjects, which will finally be published in 1749. He then sends to the journal of the Berlin Academy four more papers, one of which is specifically devoted to lunar theory. Knowing that he is a latecomer compared with Clairaut and Euler, who both had begun to work on the subject well before, d'Alembert is exceedingly cautious. He doesn't publish anything without first consulting Euler, Cramer, or other foreign colleagues. The papers he sends to Paris and Berlin are sealed and date stamped to protect against any accusation of plagiarism. In June 1747, at the Paris academy, he finally reads an essay about planetary orbits. But he never really gets into the topic and, fearing Clairaut's critics, deliberately avoids revealing the core of his method.

We have now met the major actors in the controversy. First is an attacking Clairaut who proposes to modify Newton's often-verified inverse-square law. By doing so, he is inevitably the target of much criticism—above all from England and the "straight" Newtonians. Second is Euler, who plays the unimpressed elder already in the know. The third is the timid and cautious d'Alembert, eager to acquire the personal glory he would receive for finding a lunar theory in agreement with observation, but knowing that he will have to outplay both Clairaut and Euler.

The plot twist

The situation completely changes in the spring of 1749. Much to the surprise of his colleagues, Clairaut announces in letters to his friends and in a paper read on 17 May before the Paris academy that he has found a way to reconcile the motion of the Moon and Newton's attraction law. He gives a long account of his retraction in a 3 June letter to his friend Cramer, part of which is reproduced in figure 5. In the letter, Clairaut reveals that his new realization came to him only six months after he sent his revolutionary treatise to England, to Italy, and to the famous Bernoulli family in Basel. He continues (translations here and following are mine):

> As it was very important to me that nobody should forestall me in this matter, I sent a sealed parcel to London, which enclosed my new result, and I urged Mr. Folkes [the president of the Royal Society] not to open it until I asked him to do so. And I used the same arrangement here at the [Paris] Academy. My intention was thereby to prevent anybody from showing off, saying he had corrected me, and to give me more time before pronouncing my retraction, so I could complete the calculations that led me to it.

Showing the qualities of a master strategist, Clairaut reverses the relations of power. From challenger contesting an established law and, for some contemporaries, the whole Newtonian paradigm, Clairaut repositions himself as the paradigm's defender and maintainer by affirming he has explained one of its anomalies: the motion of the Moon's apsides. His result, which he keeps secret at the time, is based on his discovery that some previously neglected higher-order perturbative terms were actually important.

Not knowing the mathematical basis of Clairaut's retraction, d'Alembert feels compelled to revoke his own explanation of lunar motion in front of the Paris academy—on the same day Clairaut announces his new result. In June 1748 d'Alembert had already written to Maupertuis, the president of the Berlin Academy, to recall all his astronomical articles but the most elementary one, which merely gives solutions to a wide range of astronomical problems. Eventually he revises his lunar theory and, in 1749, publishes it in the *Memoirs* of the Paris academy, which also contains Clairaut's paper. But even then, he is cautious, waiting for Clairaut's next move before doing anything: In a letter to Cramer written on Christmas Day 1748 he writes that his treatise "would maybe already be in print, if it wasn't for Mr. Clairaut's paper on the

Figure 4. Jean le Rond d'Alembert (1717–83). This pastel was painted by Maurice-Quentin de La Tour circa 1753. (Courtesy of the Musée Antoine Lécuyer, Saint-Quentin, France.)

Moon. I'm waiting for him to finish, to see if we agree."

By the end of the 1740s, neither Euler nor d'Alembert can reconcile the motion of the Moon's apsides with Newton's inverse-square law, as Clairaut claims to have done. D'Alembert even postulates an additional lunar magnetic force to explain the orbital irregularities, whereas Euler continues to think that the inverse-square law needs to be corrected. Fearing the loss of the priority dispute and the quest for glory in search of an accurate lunar theory, d'Alembert and Euler will employ new strategies; the sealed letters, cautiousness, and deliberate delay we have seen are only the beginning.

Positions of influence

To improve his weak position in the competition, d'Alembert exploits his standing as a chief editor of the French encyclopedia. Published between 1751 and 1772, with later supplements and revisions, the encyclopedia is one of the most ambitious and advertised publication projects of the 18th century. In three articles—"Apogee," "Apsis," and "Attraction"—d'Alembert gives his own vision of the problem of the apsidal motion. In the last of those articles he concludes, "only the Moon's apsides motion seemed for some time irreconcilable with Newton's system, but this matter is not decided at the time we write these lines; and I think, I can assure, that the Newtonian system will honorably come out."

Near the end of the article, d'Alembert announces an imminent publication on the subject. His statement is intriguing because at the time he doesn't actually have a solution that explains the apsidal motion within the framework of Newton's system. Furthermore, although d'Alembert finishes his *Recherches sur différents points importants du système du monde* between October 1750 and January 1751, he won't publish that work until 1754. By deliberately suggesting in a reference book addressed to a wide audience that he may have solved the problem of the apsidal motion before anybody else, d'Alembert is trying to ensure that posterity's verdict on the controversy will be in his favor.

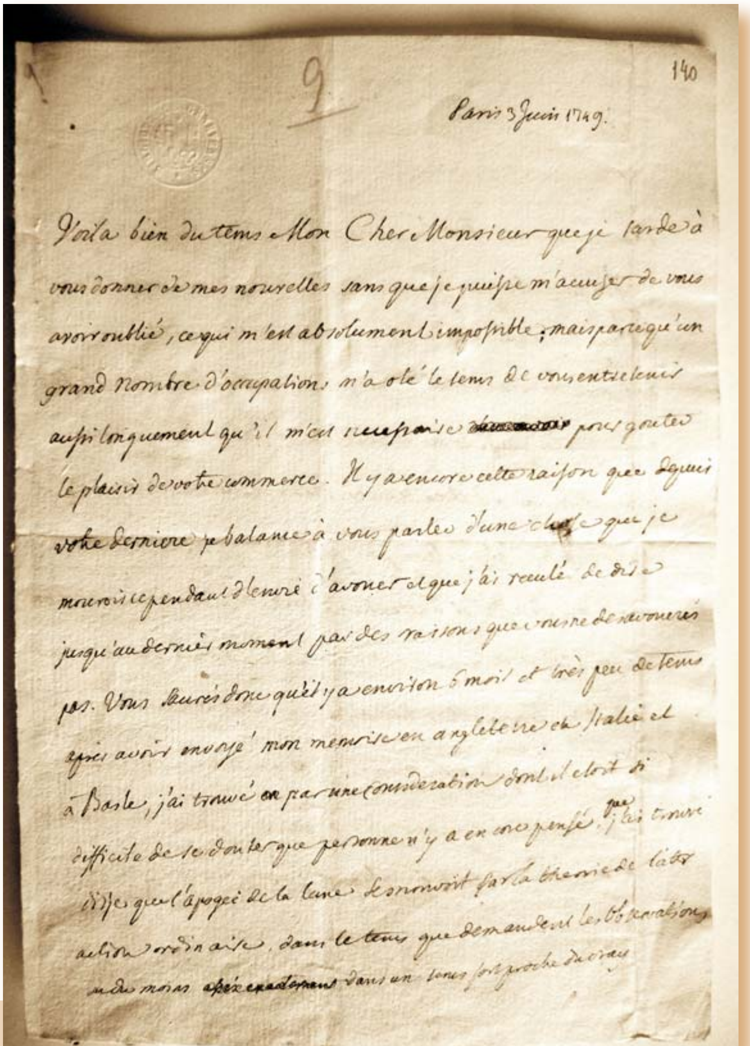
Euler's strategy is quite different. Clairaut has intentionally left him in complete ignorance of the May 1749 retraction, which Euler first hears about through an account of the young astronomer August Nathanael Grischow, who attended Clairaut's talk at the Paris academy. Euler immediately reactivates his correspondence with his French colleague, asking for Clairaut's new method. But Clairaut's answer, dated 21 July 1749, is monosyllabic, divulging almost nothing but assuring Euler that Clairaut will soon publish his computation. Clairaut keeps Euler uninformed; he doesn't even write any new letters until January 1750.

Meanwhile, the Swiss scholar loses his patience. Still influential in the Imperial Academy of Sciences in Saint Petersburg, Russia, which he had left for Berlin in 1741, he writes to its secretary, Johann Daniel Schumacher, and its chancellery assessor, Grigorij Nikolajevic Teplov, to propose the following question for the academic prize of 1750: "Does every inequality that we observe in the motion of the Moon accord with the Newtonian Theory or not?" As soon as the academy accepts his question, Euler writes to both Clairaut and d'Alembert, encouraging them to participate. As one of the judges who would examine the treatises, he has thus found a way to have a look at the methods of his competitors before anybody else.

Unfortunately for Euler, d'Alembert decides not to participate. Not only is he fully occupied with finishing the first volume of the encyclopedia, but he also fears competing with Clairaut. D'Alembert might also have guessed that Euler would evaluate the entries. Earlier, when d'Alembert entered a hydrodynamics paper in the Berlin Academy Prize compe-

A note on sources

This article makes extensive use of the records of the Academy of Sciences in Paris and its periodical *Histoire de l'Académie Royale des Sciences avec les Mémoires de Mathématique et de Physique*; both can be accessed via the Gallica online digital library at http://gallica.bnf.fr. Another key resource was the journal of the Berlin Academy of Sciences, *Histoire de l'Académie Royale des Sciences et des Belles-Lettres de Berlin*, available at http://bibliothek.bbaw.de/bibliothek-digital/digitalequellen/schriften/#A2. The online Euler Archive (http://www.math.dartmouth.edu/~euler) is an extensive inventory (and more) that includes most of the works of Leonhard Euler quoted in the article. For more than 100 years, the Euler Commission of the Swiss Academy of Sciences has been publishing Euler's complete works with extended introductions and annotations. The fourth and final series of the *Opera Omnia*, currently in production, contains much of the cited Euler correspondence. For an inspiring and more mathematical analysis of the controversy discussed in this article, see Michelle Chapront-Touzé's introduction to Jean le Rond d'Alembert's *Premiers textes de mécanique céleste 1747–1749* (CNRS Editions, 2002), in particular pages xxii–lxxii.



Paris 3 Juin 1749.

Voila bien du tems Mon Cher Monsieur que je tarde à vous donner de mes nouvelles sans que je puisse m'accuser de vous avoir oublié, ce qui m'est absolument impossible; mais parceque un grand nombre d'occupations m'a oté le tems de vous entretenir aussi longuement qu'il m'est necessaire pour gouter le plaisir de votre commerce. Il y a encore cette raison que depuis votre derniere je balance à vous parler d'une chose que je mourois cependant d'envie d'avouer et que j'ai reculé de dire jusqu'au dernier moment par des raisons que vous devinerés [illegible]. Vous saurés donc qu'il y a environ 5 mois et très peu de tems après avoir envoyé mon memoire en angleterre et italie et à Basle, j'ai trouvé par une consideration dont il etoit si difficile de se douter que personne n'y a encore pensé, que j'ai trouvé dis-je que l'apogée de la lune se mouvoit par la theorie de l'attraction ordinaire, dans le tems que demandent les observations au du moins dans un tems fort proche du vrai

Figure 5. In a June 1749 letter written to his friend, mathematician Gabriel Cramer, Alexis Clairaut describes how he has reconciled the motion of the Moon's apsides with Newton's law of gravitational attraction. (Courtesy of the Geneva Library, Switzerland.)

tition for 1750, Euler deemed it unworthy; thus d'Alembert may have had reason to fear Euler's partiality. Thanks to a letter from d'Alembert to Cramer, though, we know that the French mathematician actually began to work on a paper for Saint Petersburg at the beginning of 1750. But he then abandoned the project.

In February 1751 Euler finally obtains Clairaut's paper; even after a first reading, it is clear as day that the Frenchman will win the prize. Euler spends the whole of March verifying Clairaut's calculations and, inspired by the results of his competitor, resumes research of his own to explain the apsidal motion—this time in accord with the inverse-square law. His goal is to publish his lunar theory together with Clairaut's prize-winning treatise. To that end, in October 1751 he corresponds with Schumacher and asks if his essay might appear along with Clairaut's. However, he subsequently learns that the Saint Petersburg academy can publish his work in Berlin. Possibly desirous of being the first to publish on such an important matter and hence to gain the recognition of the public, he writes again to Schumacher to get his manuscript back. The letter, alas, doesn't clearly reveal whether Euler really plans to snatch away Clairaut's precedence or if he just hopes to have better control over the publication process. Unfortunately for Euler, the Berlin publication project fails and he is forced to send his essay to Saint Petersburg again. It is eventually published there in 1753, several months after Clairaut's prize-winning treatise appears in print.

Long before Clairaut's striking publication, Euler and d'Alembert recognized that their colleague was the first to have found a lunar theory that was fully consistent with Newton's system. Given that, it is amazing to see that both try to ensure that the priority dispute is decided in their favor, fighting for their own methods and to gain the support of other scholars.

D'Alembert's foreword in his *Recherches sur différents points importants du système du monde* furnishes a typical example of how they proceeded. In it, d'Alembert reconstructs his own version of the dispute. He claims to have already found the principles of his book in January 1751 and to have sent them sealed to the secretary of the Paris academy. That allows d'Alembert to maintain that he found a solution to the problem of the apsidal motion nearly nine months before the Saint Petersburg academy awarded its prize for Clairaut's essay and long before any other book was to be published on the matter. What d'Alembert doesn't acknowledge is, first, that Clairaut had already found a solution in 1749 and, second, that Euler's lunar theory was published in 1753, a year before the *Recherches*. That omission rightfully incenses Euler, who pours his heart out to his London friend Johann Kaspar Wettstein. In a letter dated 16 November 1754, Euler criticizes d'Alembert, who has affirmed that nobody outside of France has developed the lunar theory to perfection. He retorts that he has undoubtedly worked on the matter before Clairaut and d'Alembert even thought about it. Euler also claims that neither d'Alembert nor Clairaut has found a method that could deliver accurate lunar tables. The lunar-tables angle is a recurrent argument of Euler's; let us now see why.

The not-so-obvious motive

Many of the important scenes of the drama have now played out. When the curtain finally falls, all three protagonists will have proved to be somewhat successful in convincing the public that they were the first true discoverer of a lunar theory compatible with the observations. Indeed, the authors of the three mathematicians' eulogies each believed that their hero was the winner of the priority dispute. Even today, various accounts of the controversy have important differences.

But why was the fight for priority so important for our three protagonists, who were already famous scholars in the Republic of Letters? Was it really just another battle for glory? What were the real stakes underlying the problem?

The development of a lunar theory agreeing with observations was more than just a simple chapter in the history of Newton's gravitational law and its adoption in continental Europe. It had a practical importance that in the 18th century could not be undervalued: It helped sailors determine the longitude of their ships. The issue had important economical and strategic implications. After all, knowledge of a boat's position is crucial for returning to previously visited distant islands or harbors to establish trade routes and also for planning battles. The British Navy had endured terrible accidents; eventually the British Parliament enacted the famous Longitude Act of 1714, which offered £20 000 to the first person who could find a method to determine the longitude within an error of half a degree.

At the time the prize was offered, no one knew how to build a clock that accurately gave time in the rough environment of a sailing ship. Most 18th-century scholars, therefore, believed in an astronomical solution to the longitude problem—as did Clairaut, Euler, and d'Alembert, who all mentioned the longitude question in their works. The idea is that the Moon and the satellites of Jupiter, as they move through the sky, are like the watch hands of a giant clock. Indeed, combining the best guess of the true position of the Moon with telescope observations taken at a given time and place was still the principal method used on ships to determine longitude. To accurately decipher a celestial clock, however, an observer needs to have tables that accurately describe the position of the celestial body.

Most of the attempts that were submitted to the Board of Longitudes in Greenwich were thus of an astronomical nature. One of them arrived in 1755 from Tobias Mayer, a young astronomer from Göttingen, Germany. The lunar tables he sent were indeed very accurate. A look at his correspondence and works reveals that his tables are based on the lunar theory of none other than Euler, who was eventually awarded £300 from the Board of Longitudes.

The morals of the drama

The debate on the lunar theory shows scientists at work, communicating or holding back their solutions, arguing for their views, and strategizing as they fight for their truth. It reveals science as a collective activity, but also one that includes disputes, polemics, and controversies that are frequently governed by arguments or motives external to science. I have emphasized the importance of personal goals such as the quest for glory, esteem, and public recognition. But the competition also had clear economic implications: A lofty reputation in the Republic of Letters meant more students who would pay for their instruction. And then there were the academic prizes, which could equate to several months' wages.

Another important factor to note is the networks in which our protagonists operated. Clairaut uses his various European colleagues and his contacts in the Royal Society in London and the Paris academy, to whom he divulges his discoveries or not depending on what serves him better. D'Alembert exploits the French encyclopedia and addresses a wider public to influence posterity's decision in his favor. Euler uses his contacts at the Saint Petersburg academy to stay informed of Clairaut's and d'Alembert's progress. He even tries to capitalize on those contacts to get his work published with or before that of his competitor Clairaut. Perhaps the key strategy for anyone embroiled in controversy is to stay informed. That's why all our protagonists reactivate their correspondence networks in the course of the dispute. Cramer, the mathematician who keeps in touch with all three rivals, stands as a central figure—an external observer who deserves much more attention than I could possibly give here.

Not least, the controversy reveals some of the strategies deployed by 18th-century scientists to propagate their views. Those include the almost unexplored practice of sending sealed and date-stamped letters or parcels to arrogate the priority of their discoveries, the exploitation of academic awards, and the tactic of transmitting information and releasing printed works at the right time and place.

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